Lecture 2

Binary Search Trees

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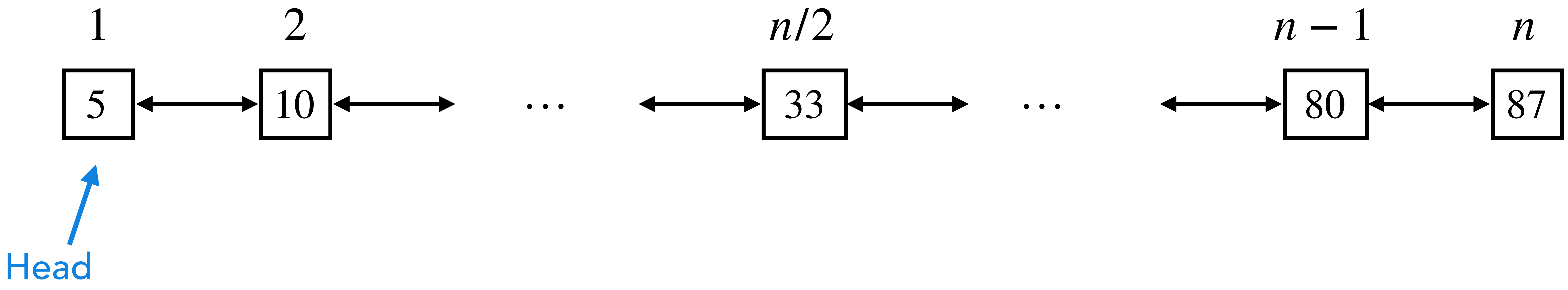
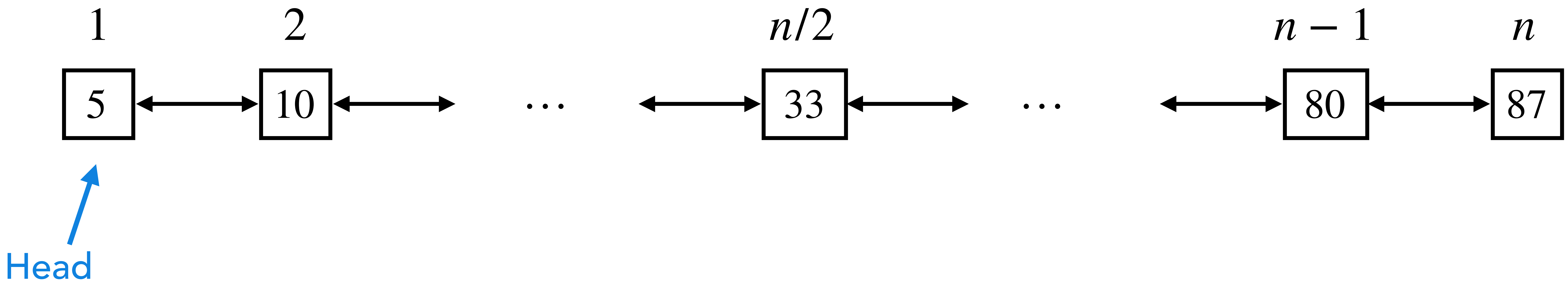
Let's first implement the operations through sorted linked list

Linked List Implementation



Linked List Implementation



	Time required in Linked list implementation
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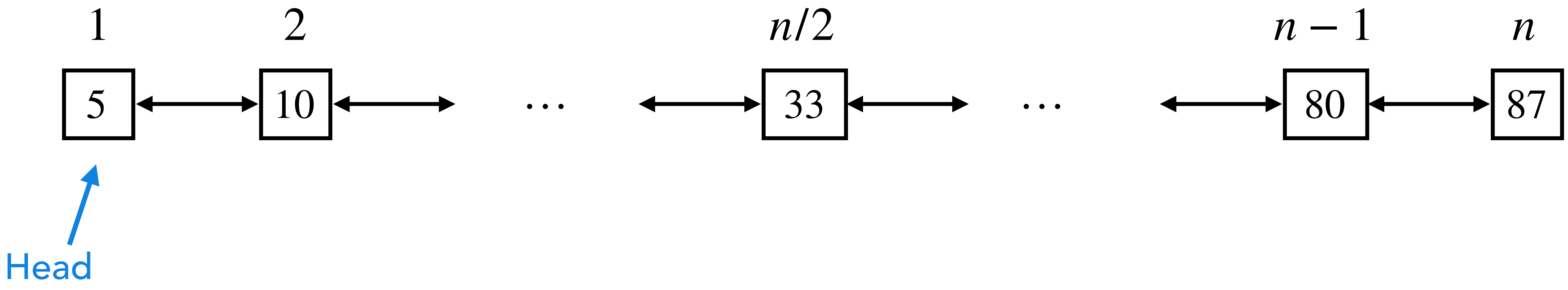

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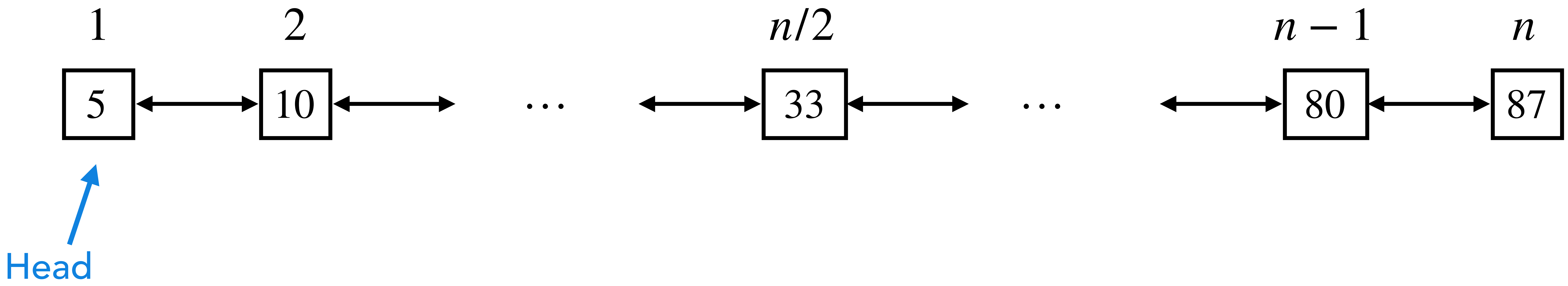

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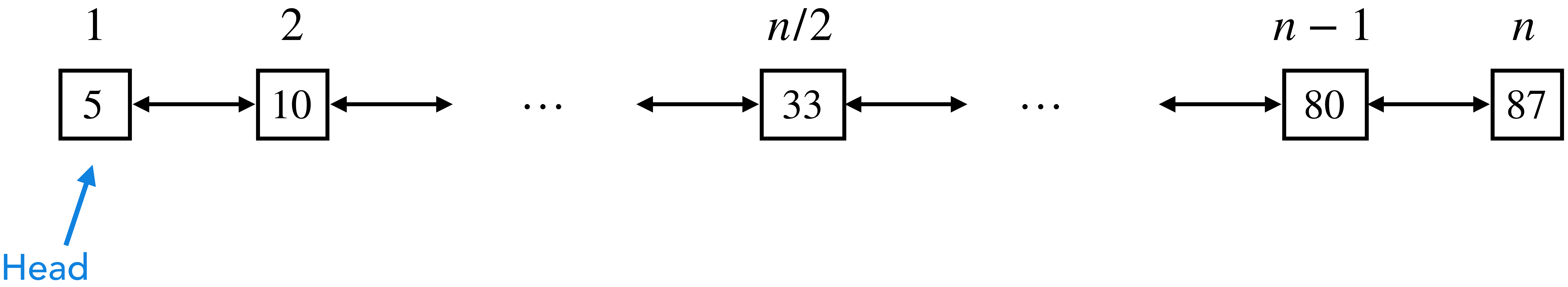

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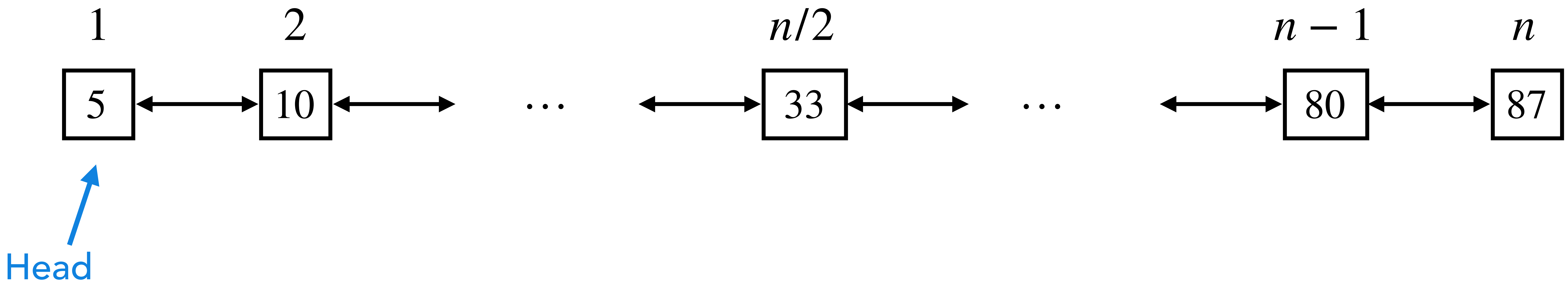

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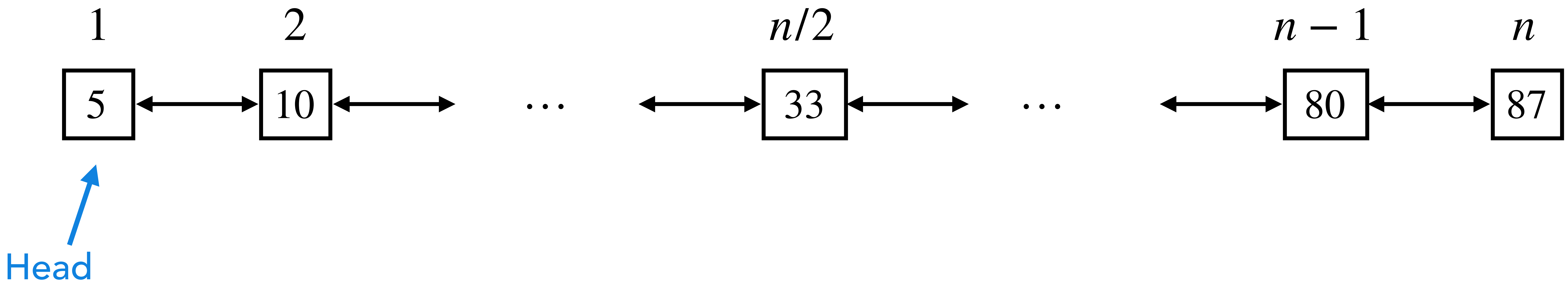

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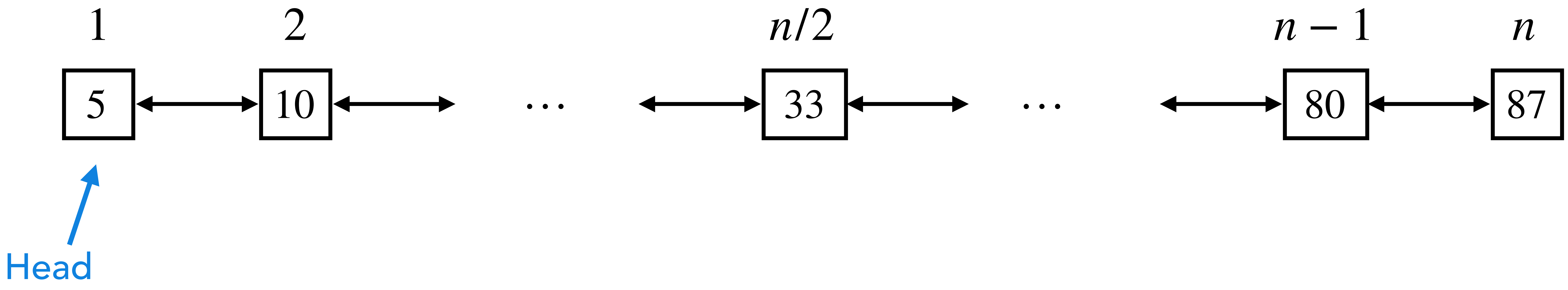

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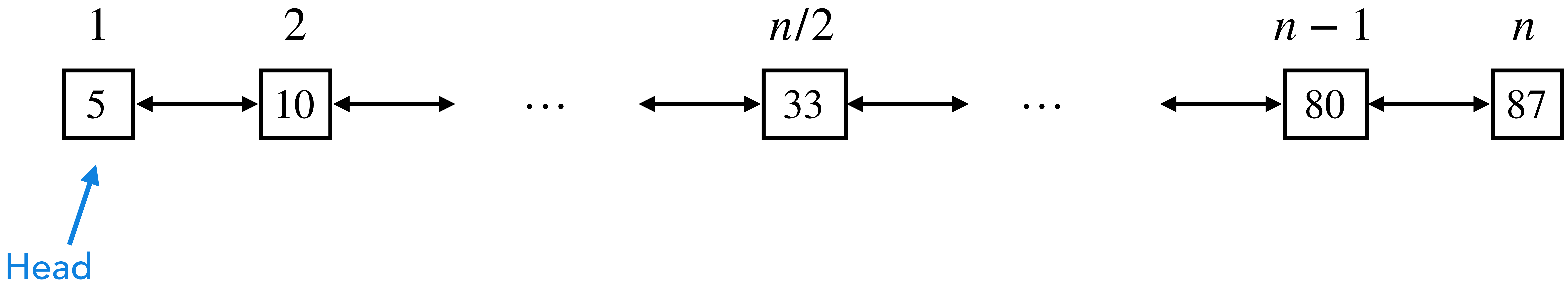
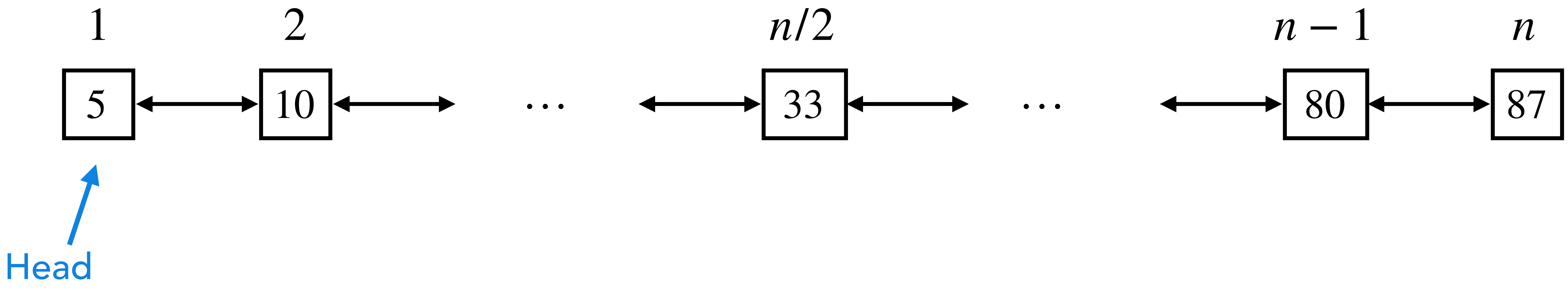
Let's try to reduce this first

Reducing Search Time in Linked Lists



Source: *DSA Slides of Surender Baswana*

Reducing Search Time in Linked Lists



Searching for a node may take at most n comparisons.

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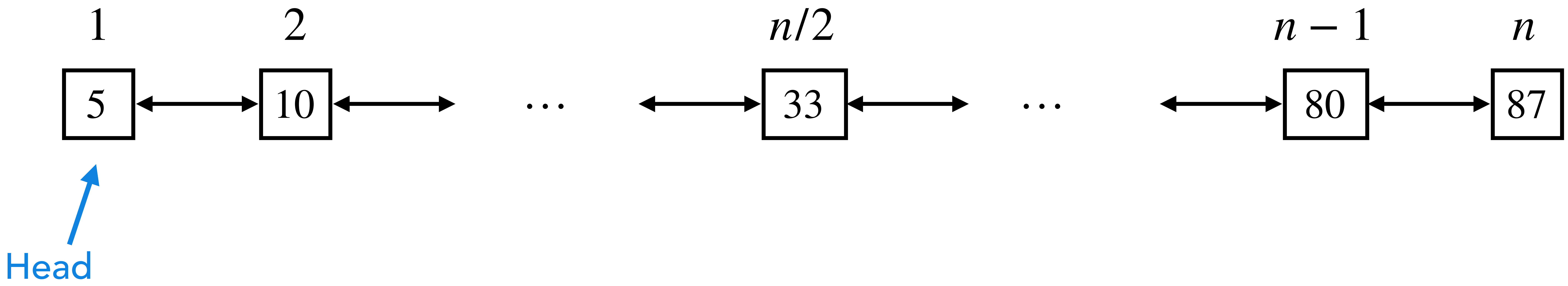

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What can be done to find a node with around $n/2$ comparisons?

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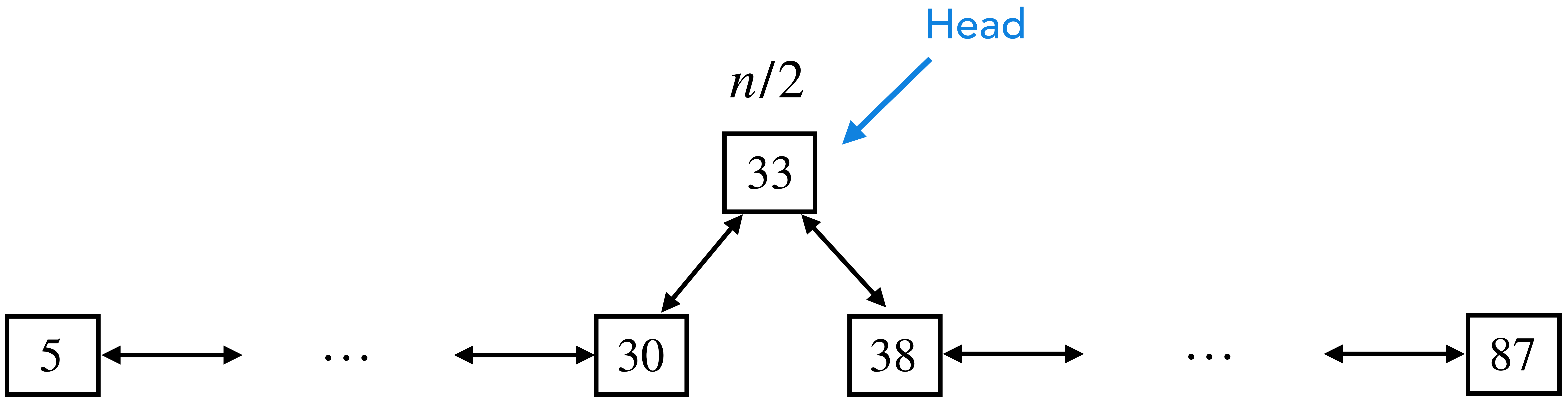

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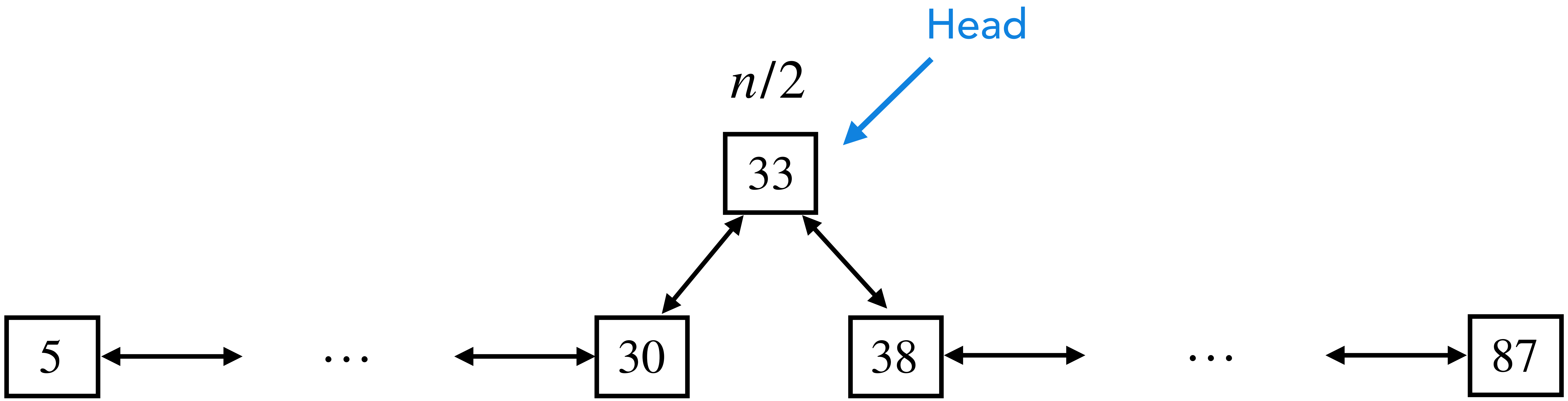

What can be done to find a node with around $n/4$ comparisons?

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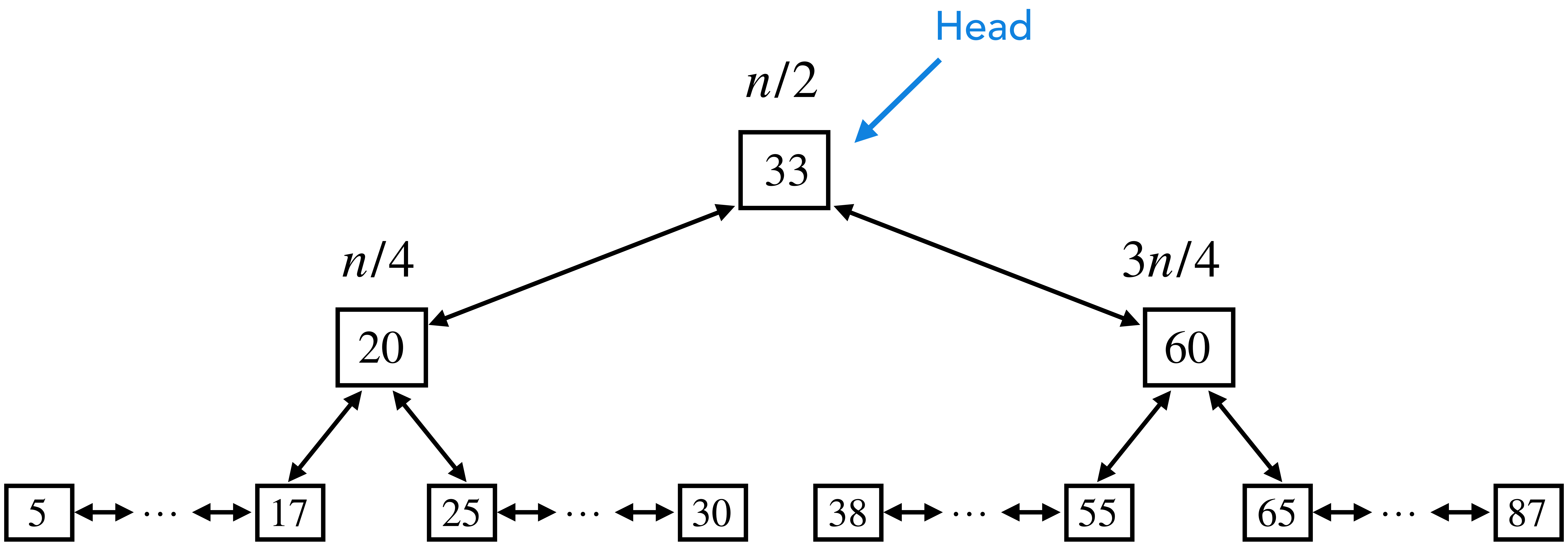
Reducing Search Time in Linked Lists



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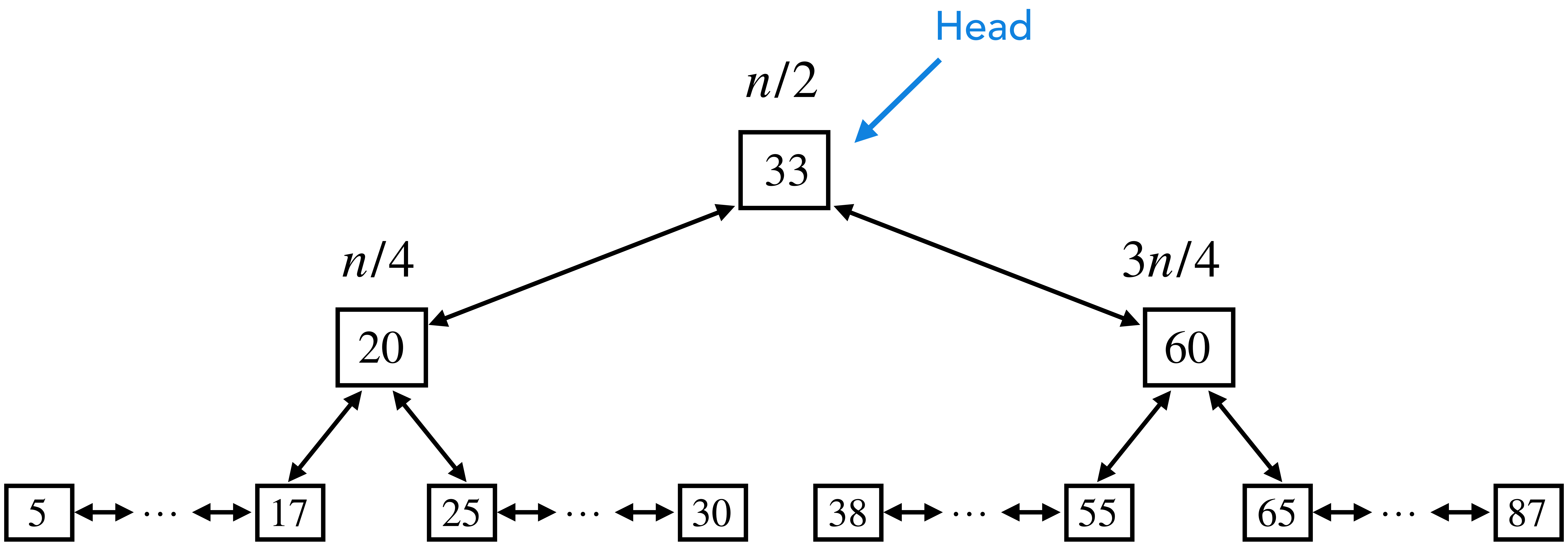

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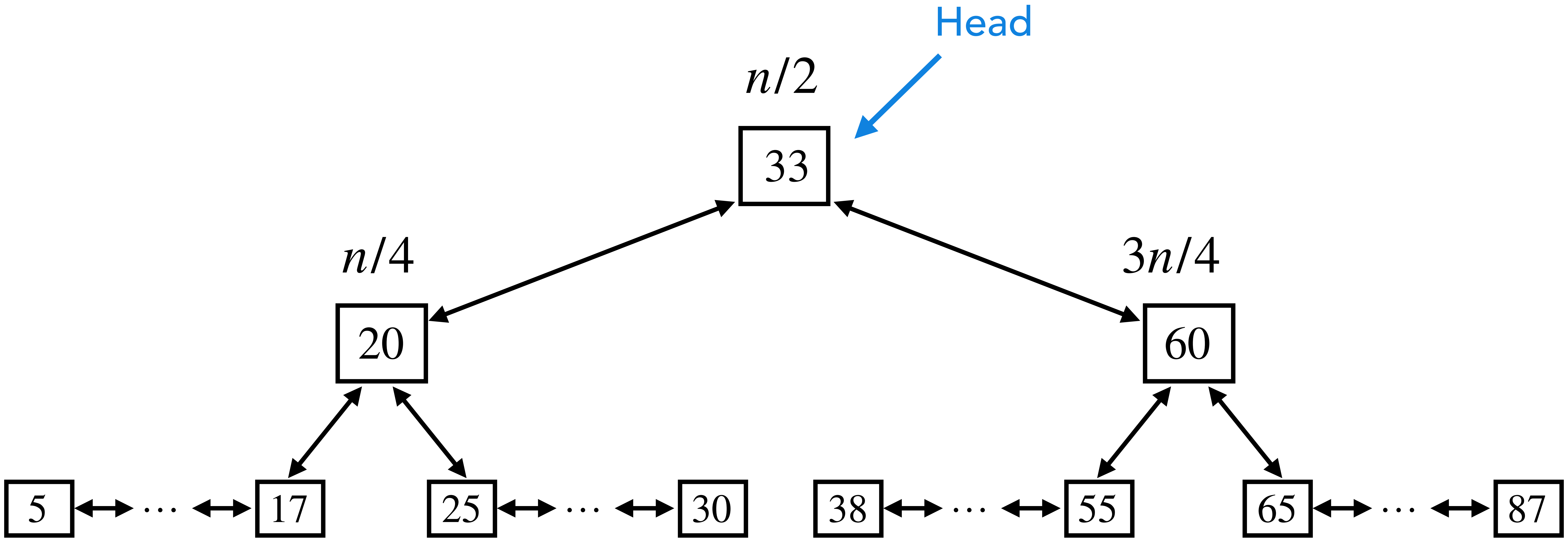

What will happen if we continue like this?

We get a binary search tree

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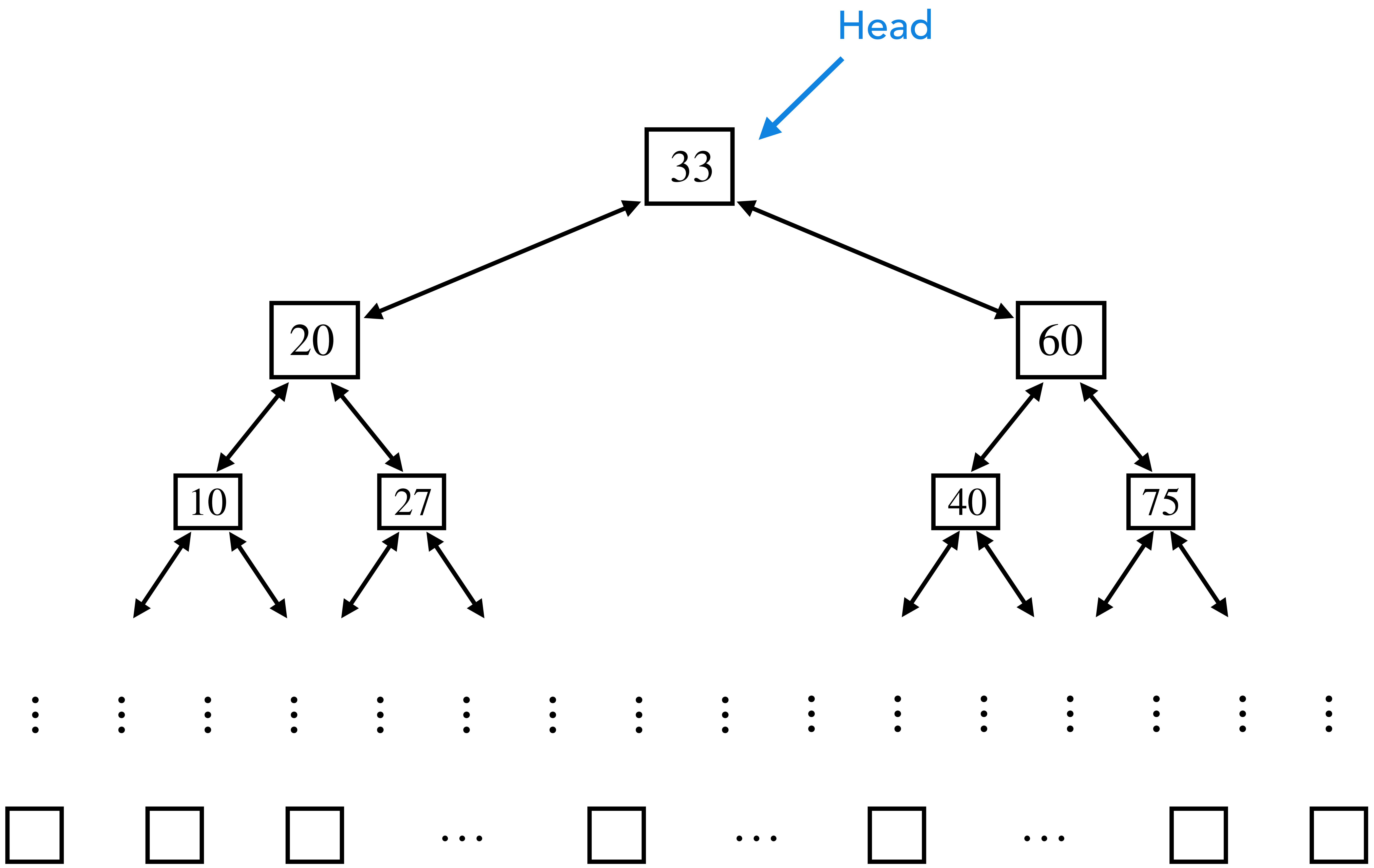
Binary Search Tree



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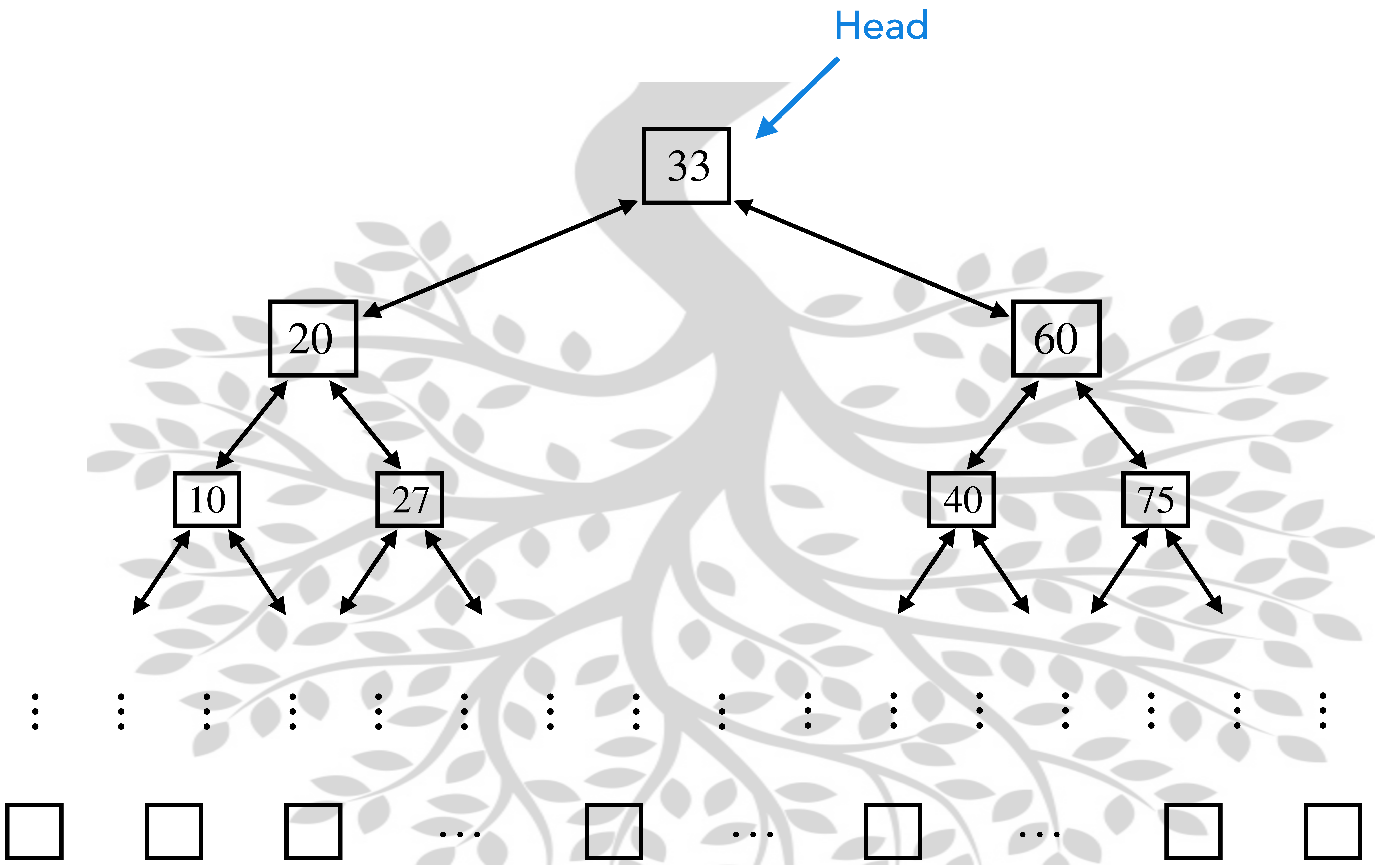

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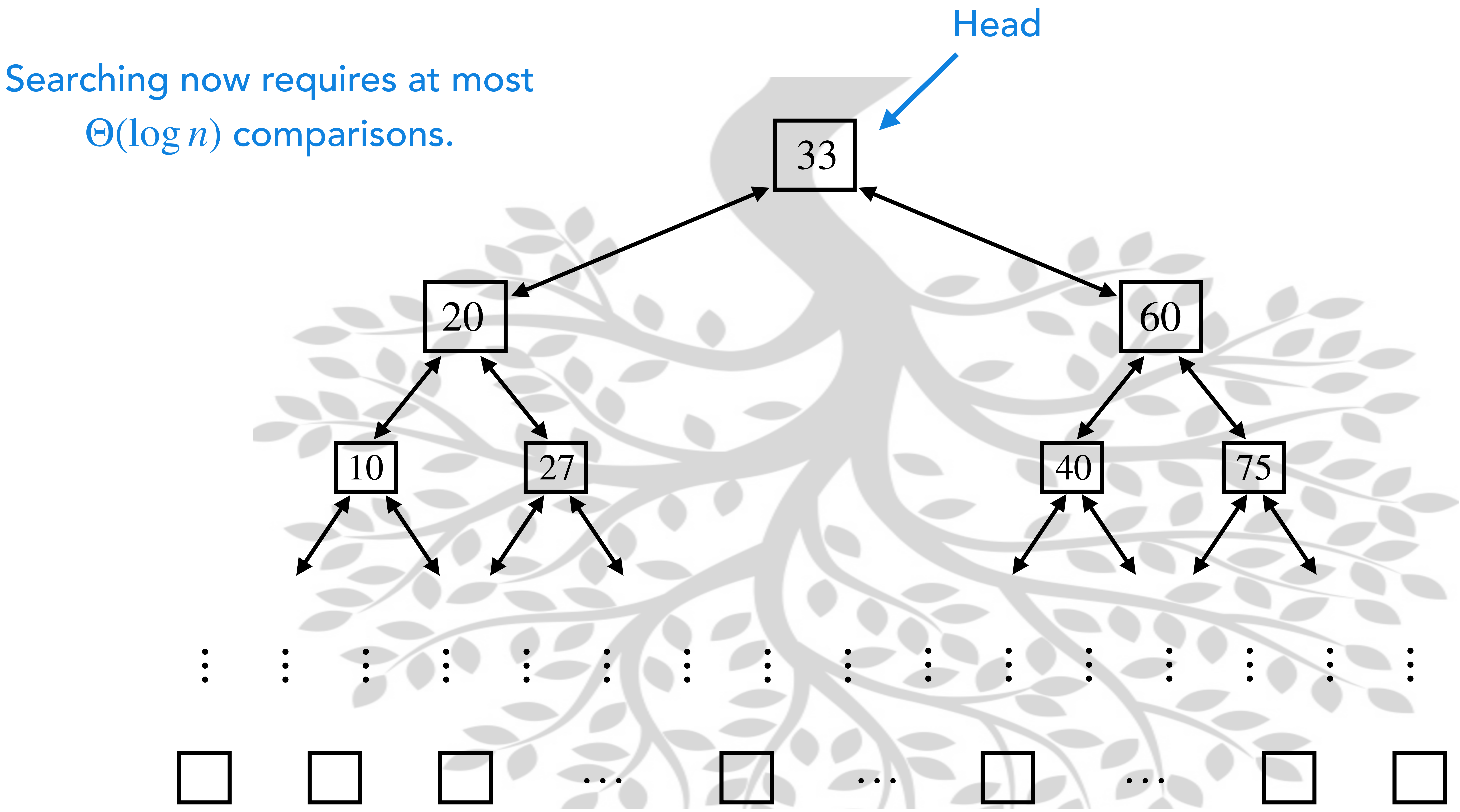

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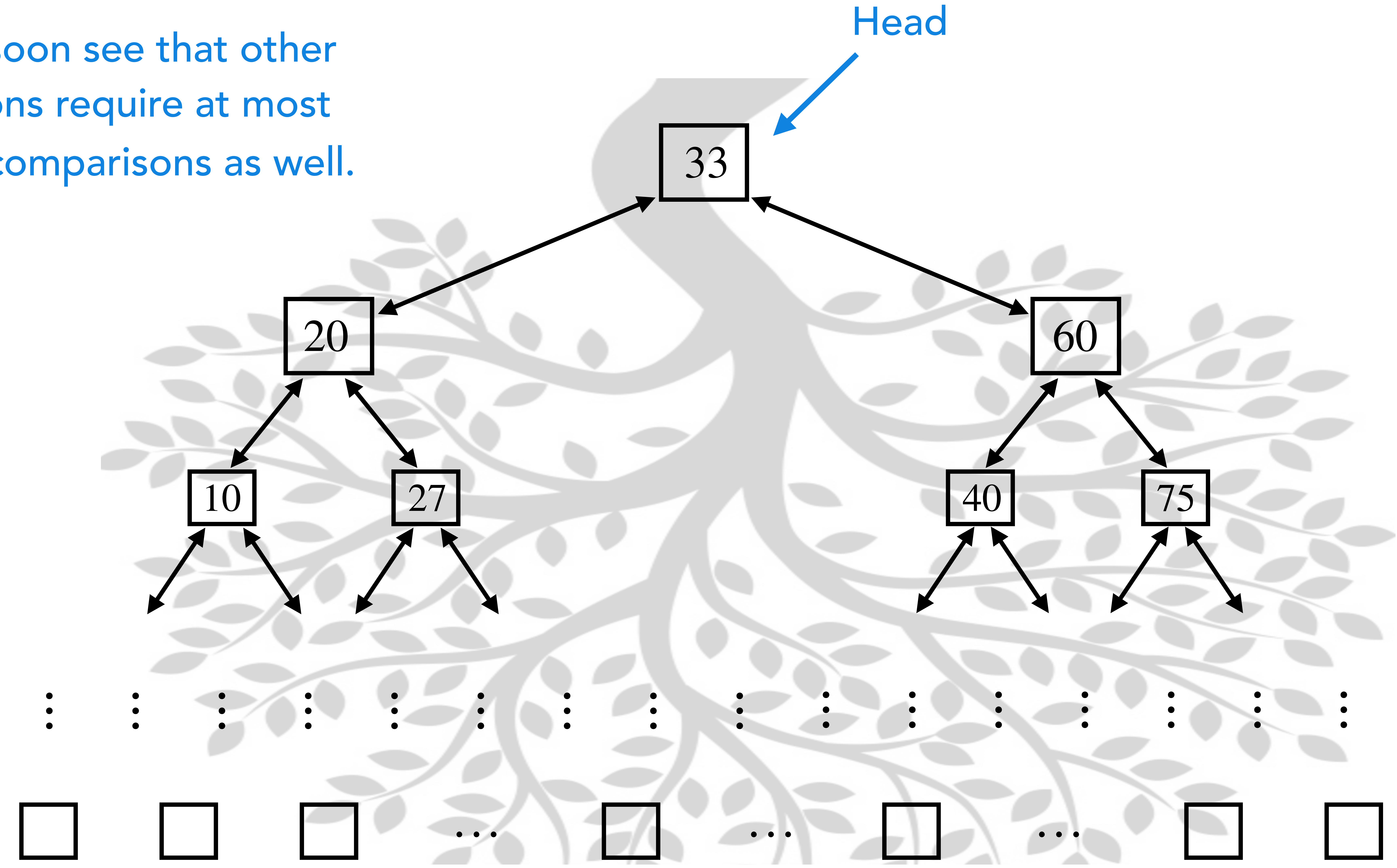
Binary Search Tree

We will soon see that other operations require at most $\Theta(\log n)$ comparisons as well.



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DIY

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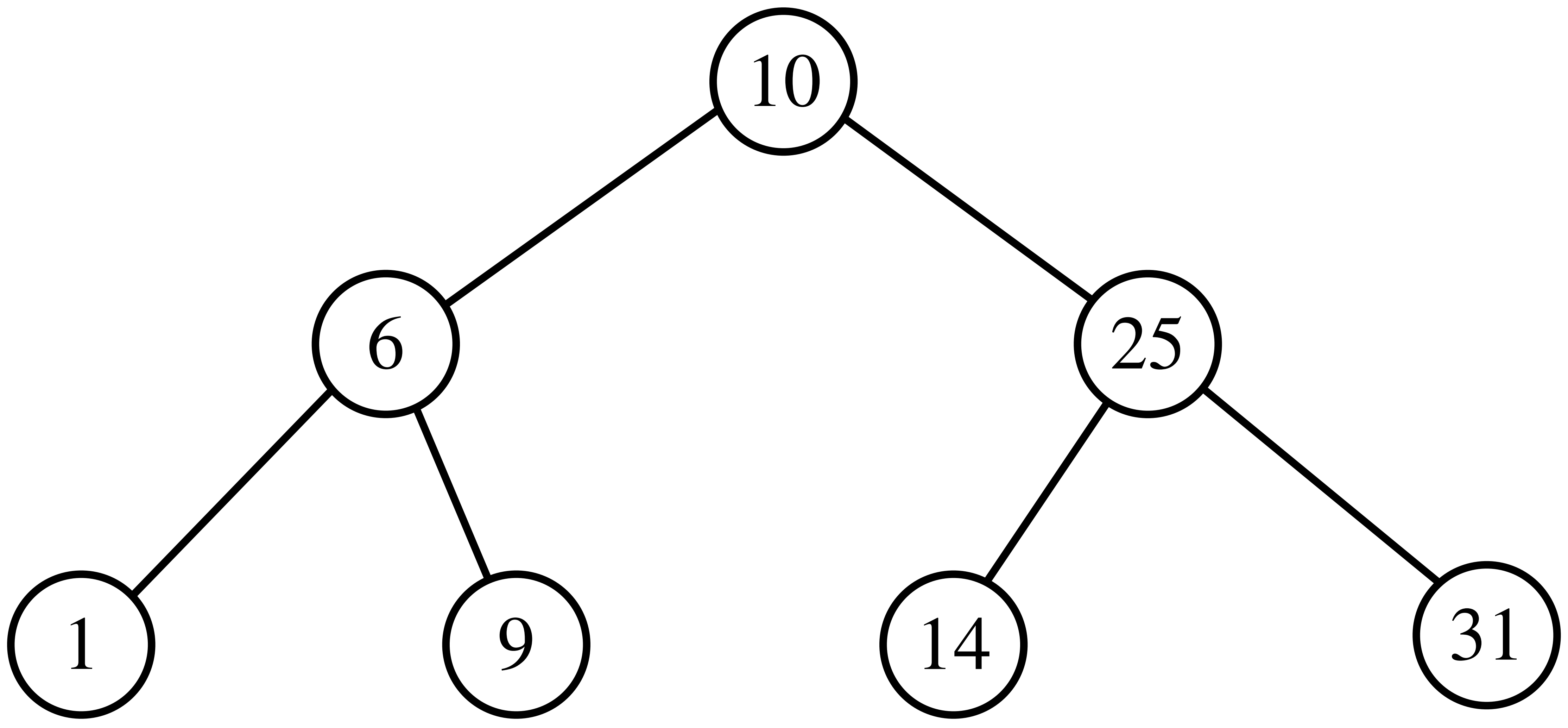

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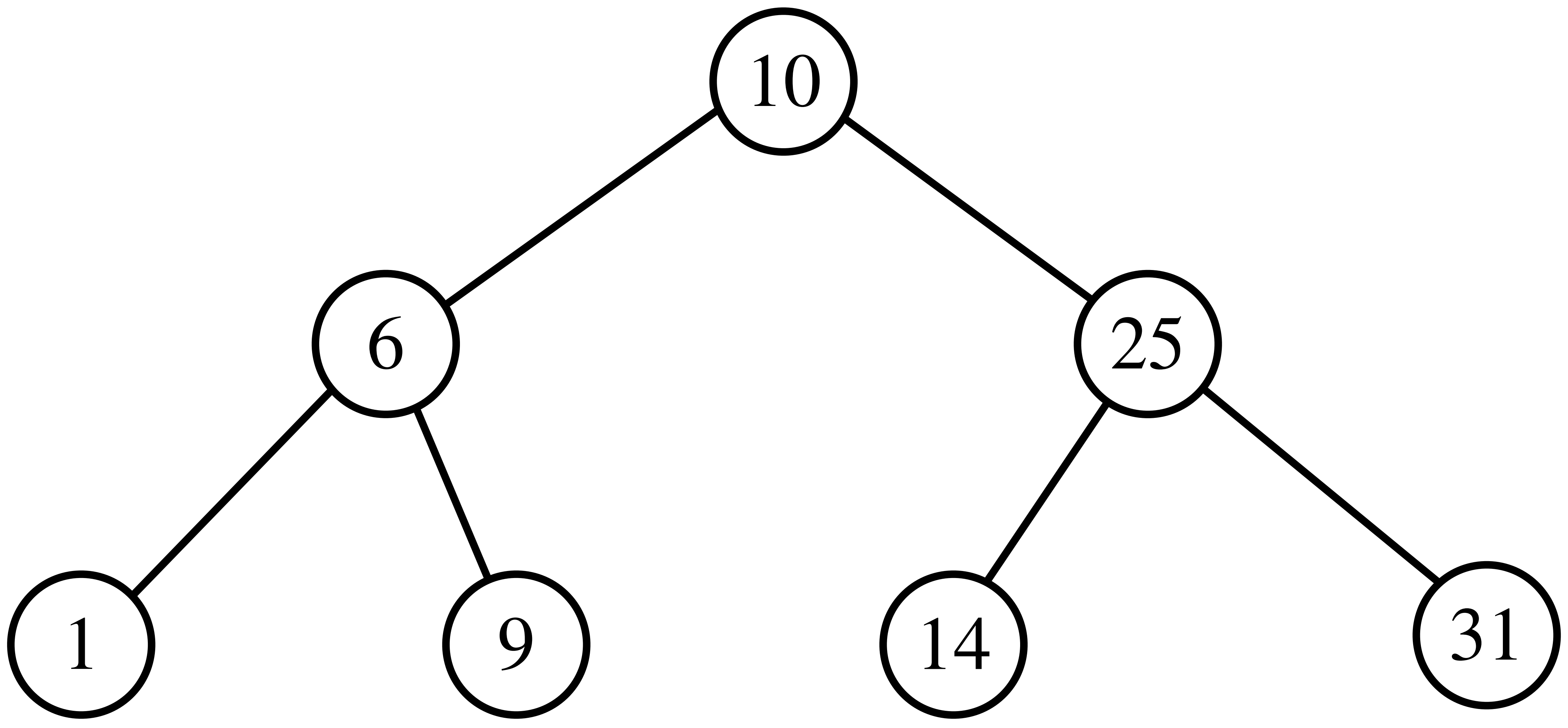

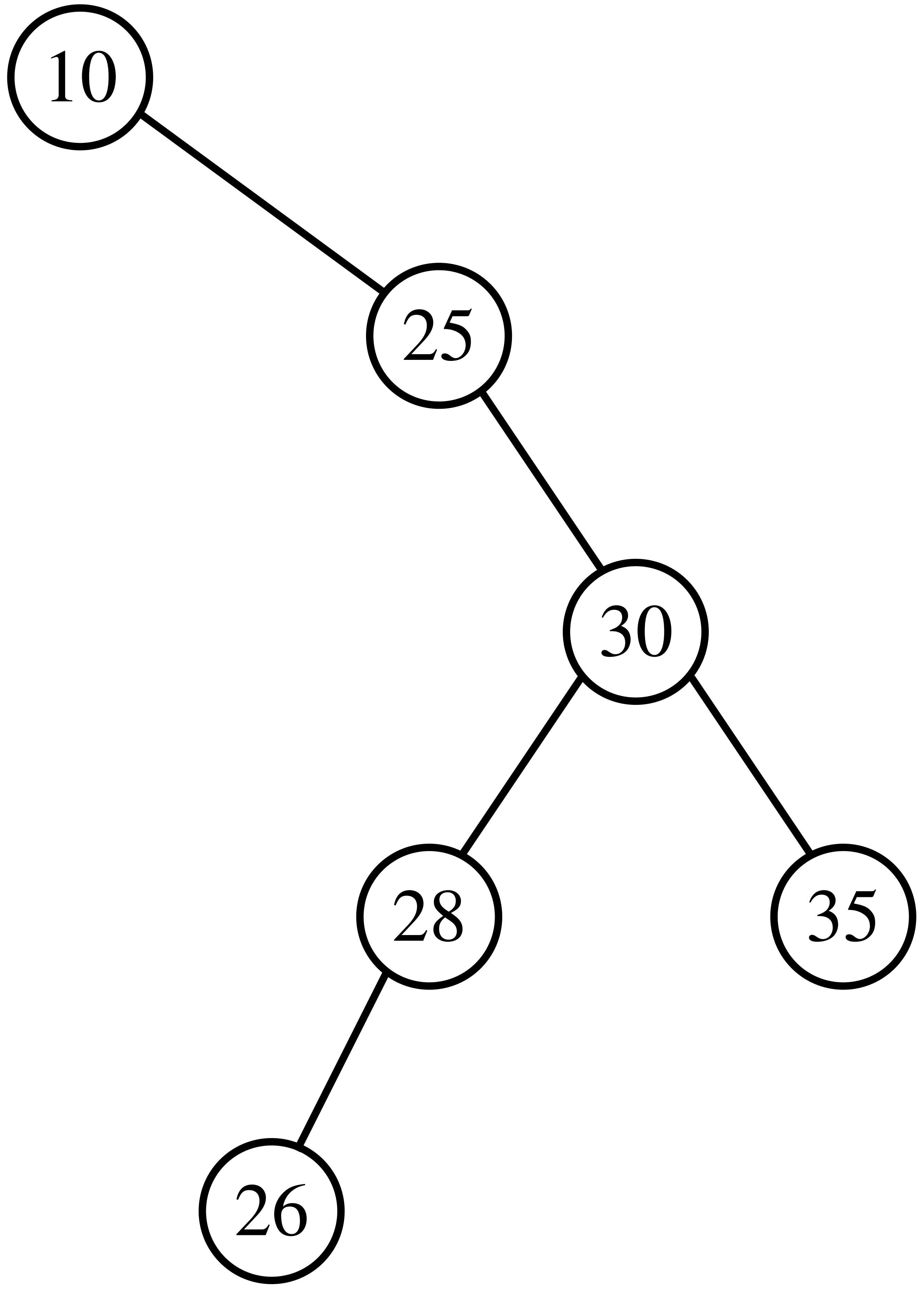

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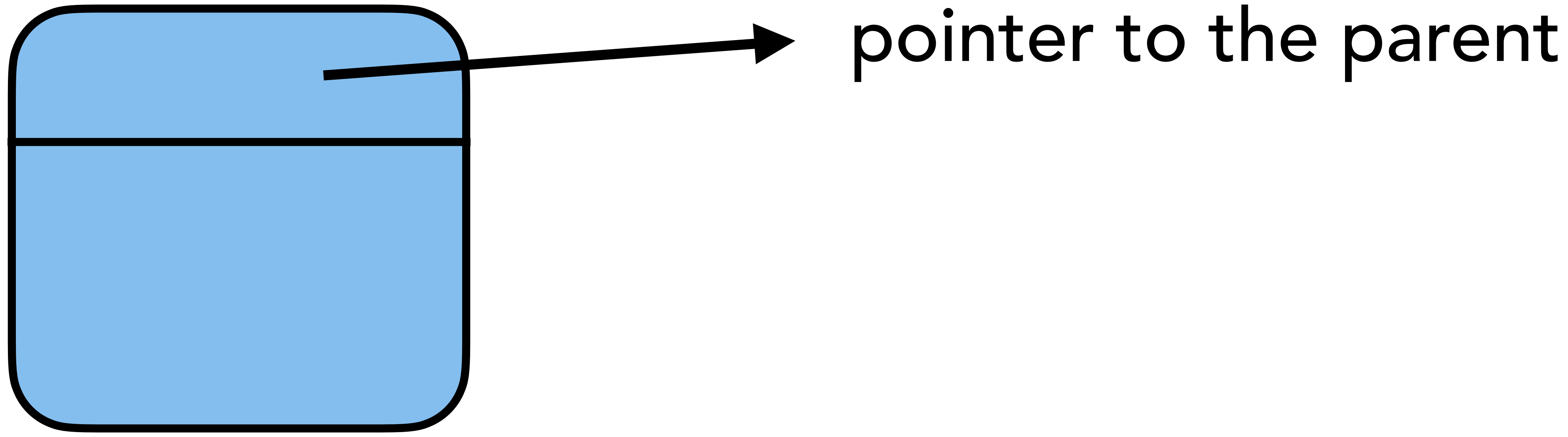

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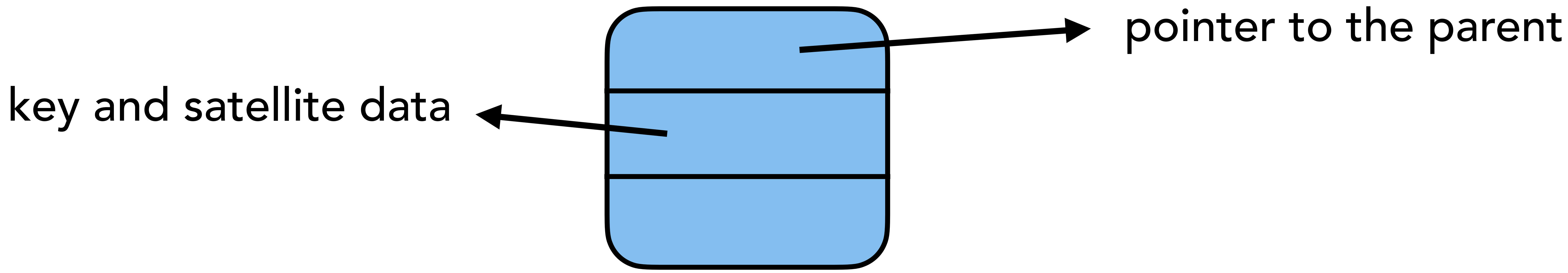

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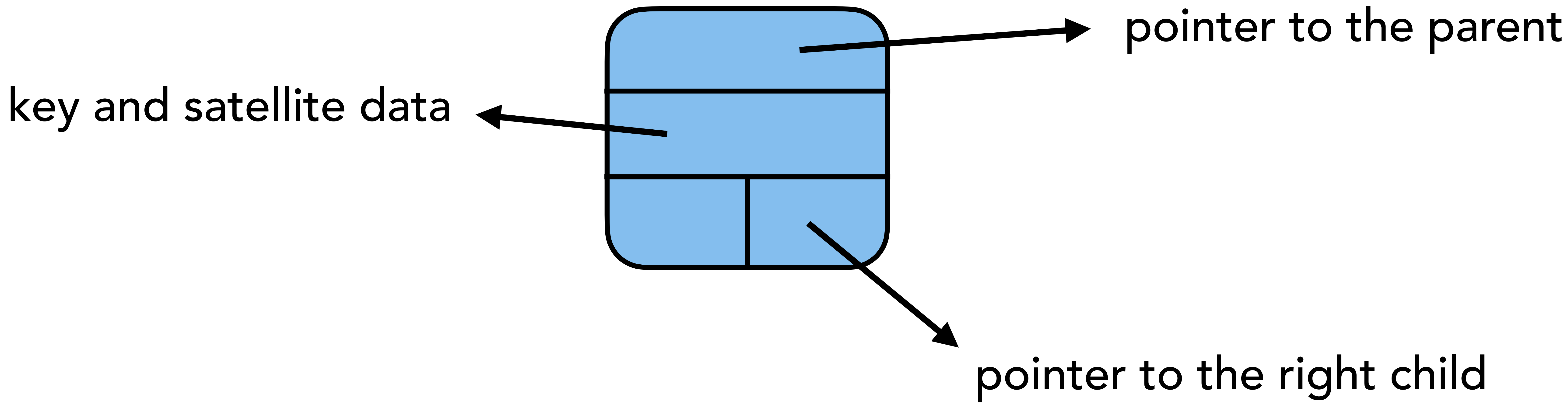

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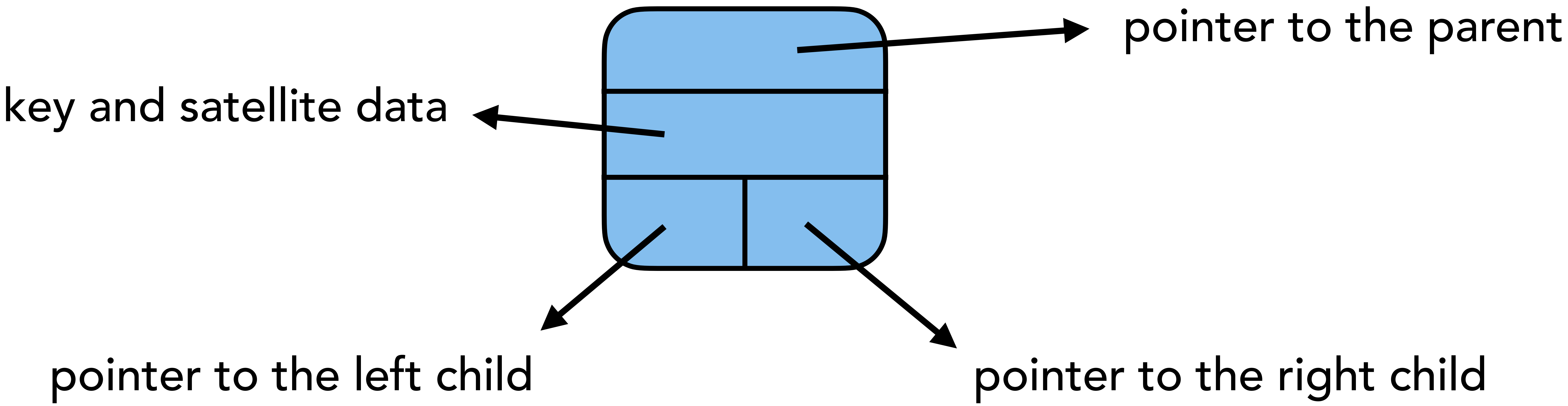

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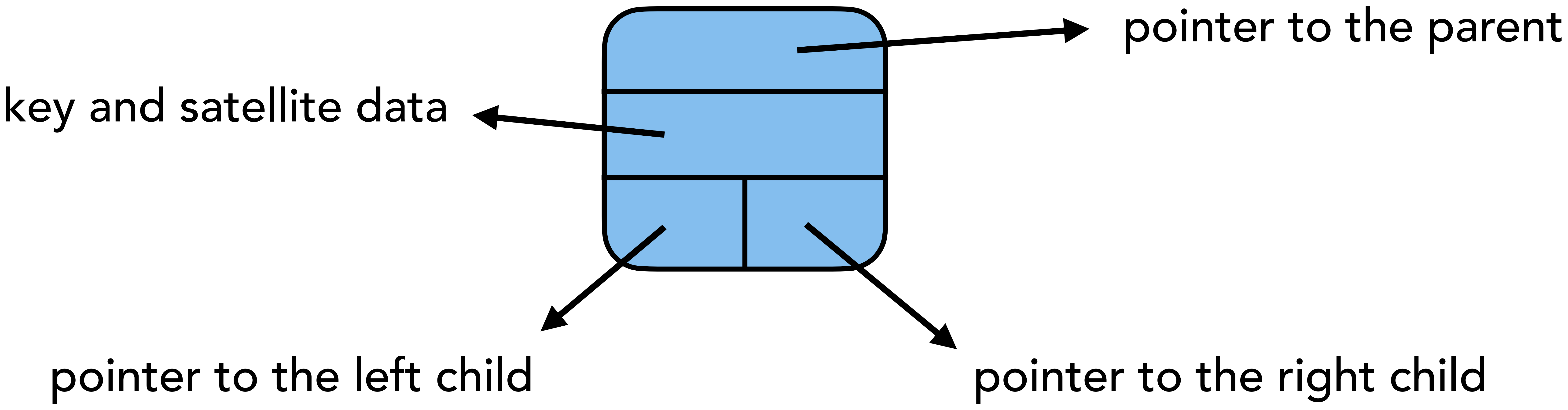

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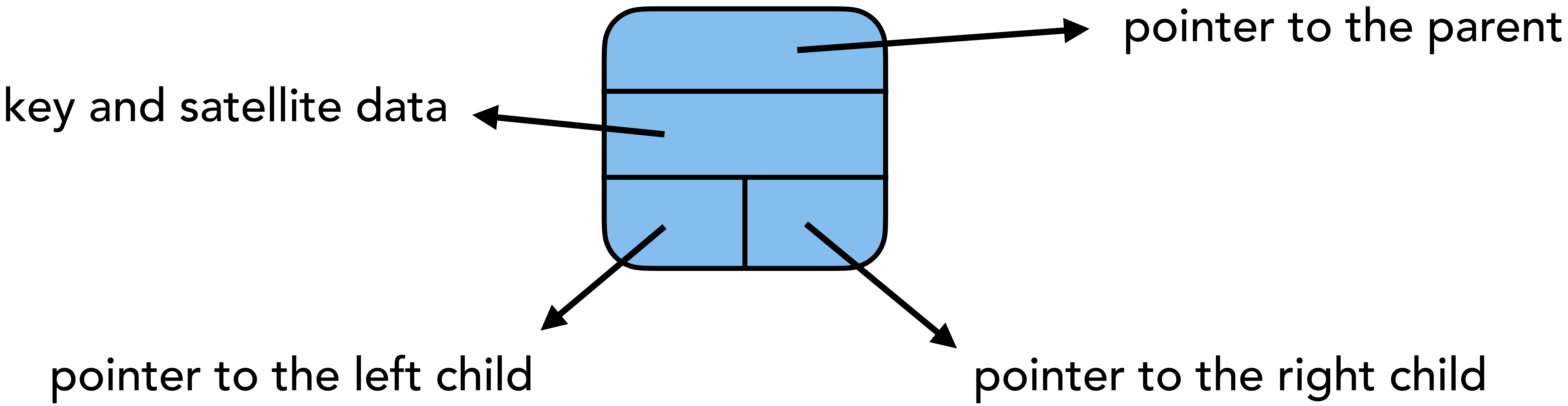

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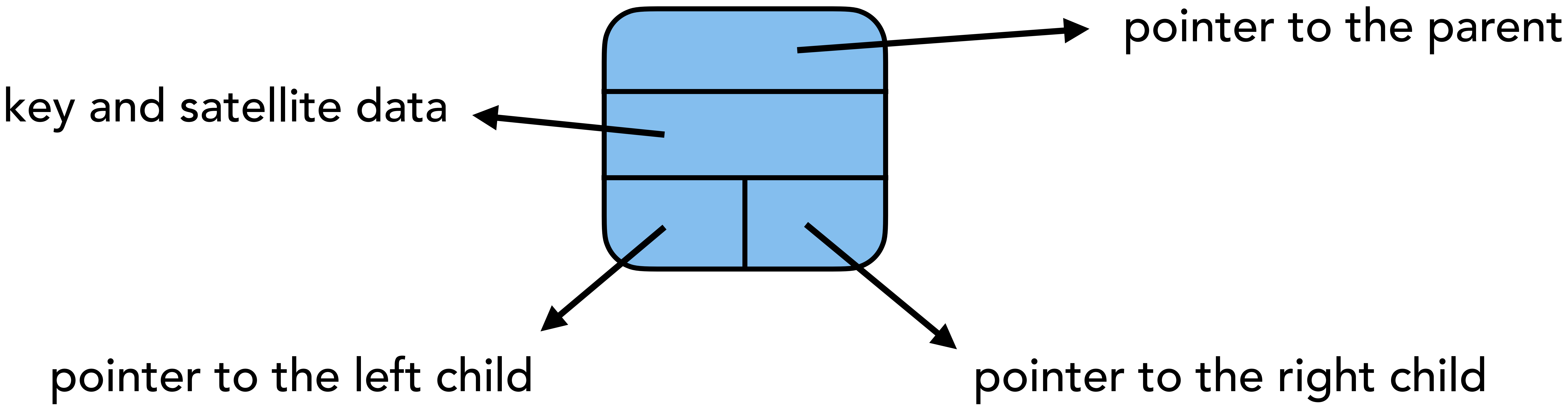

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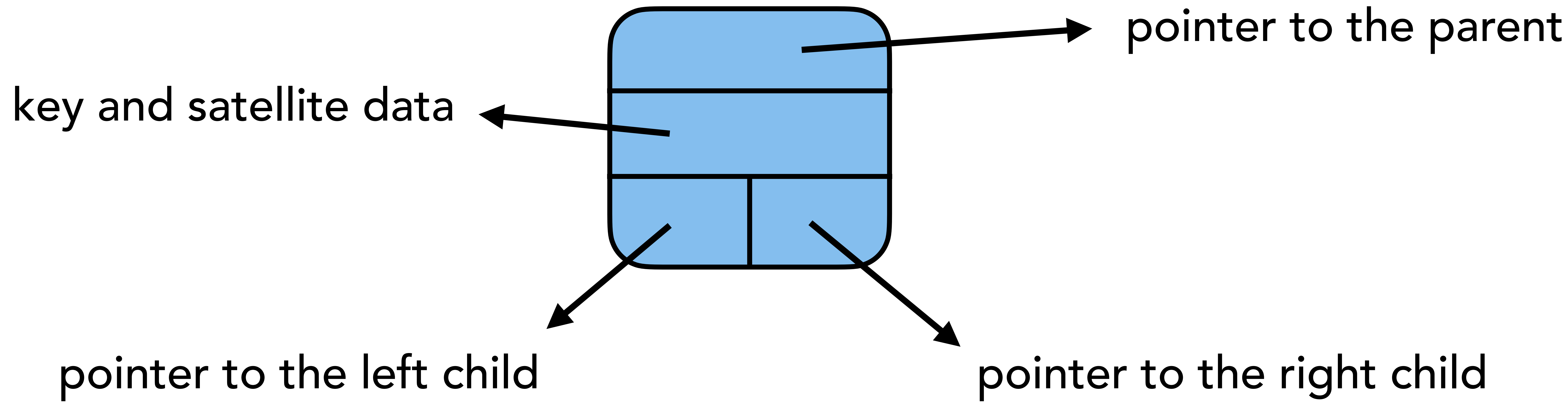

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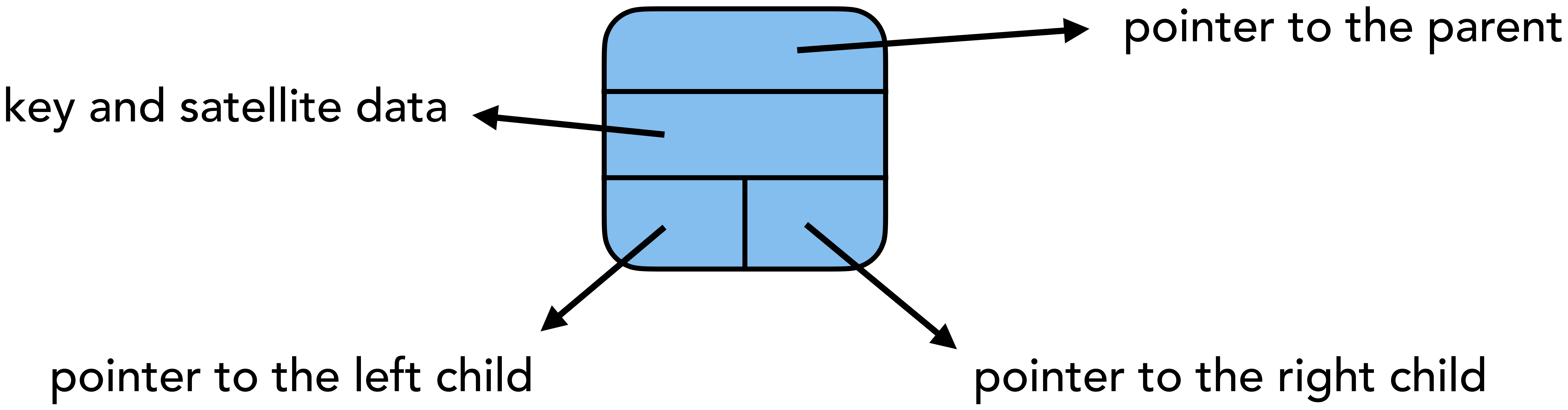

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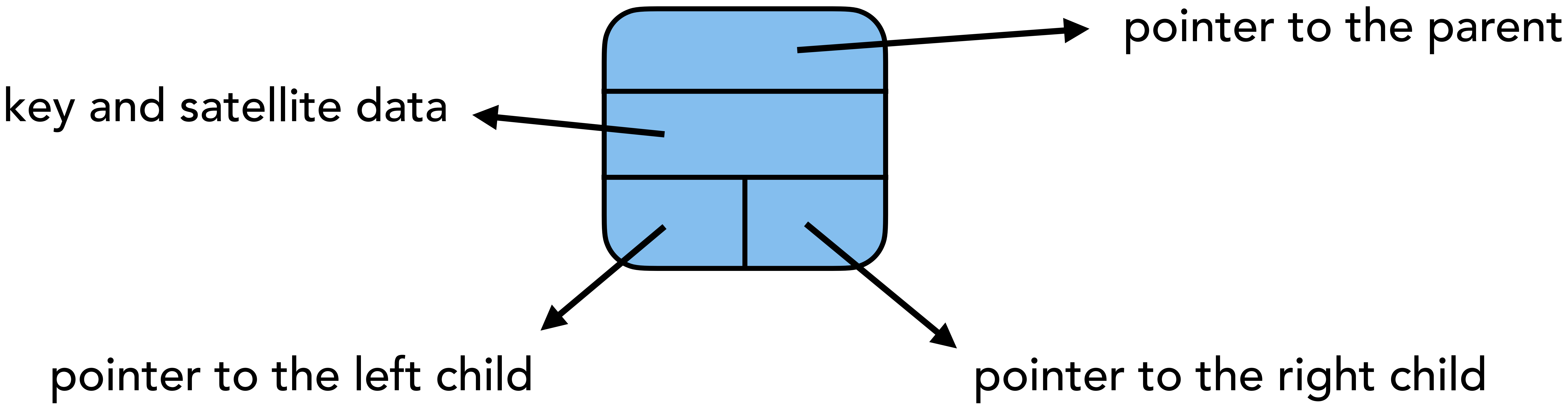

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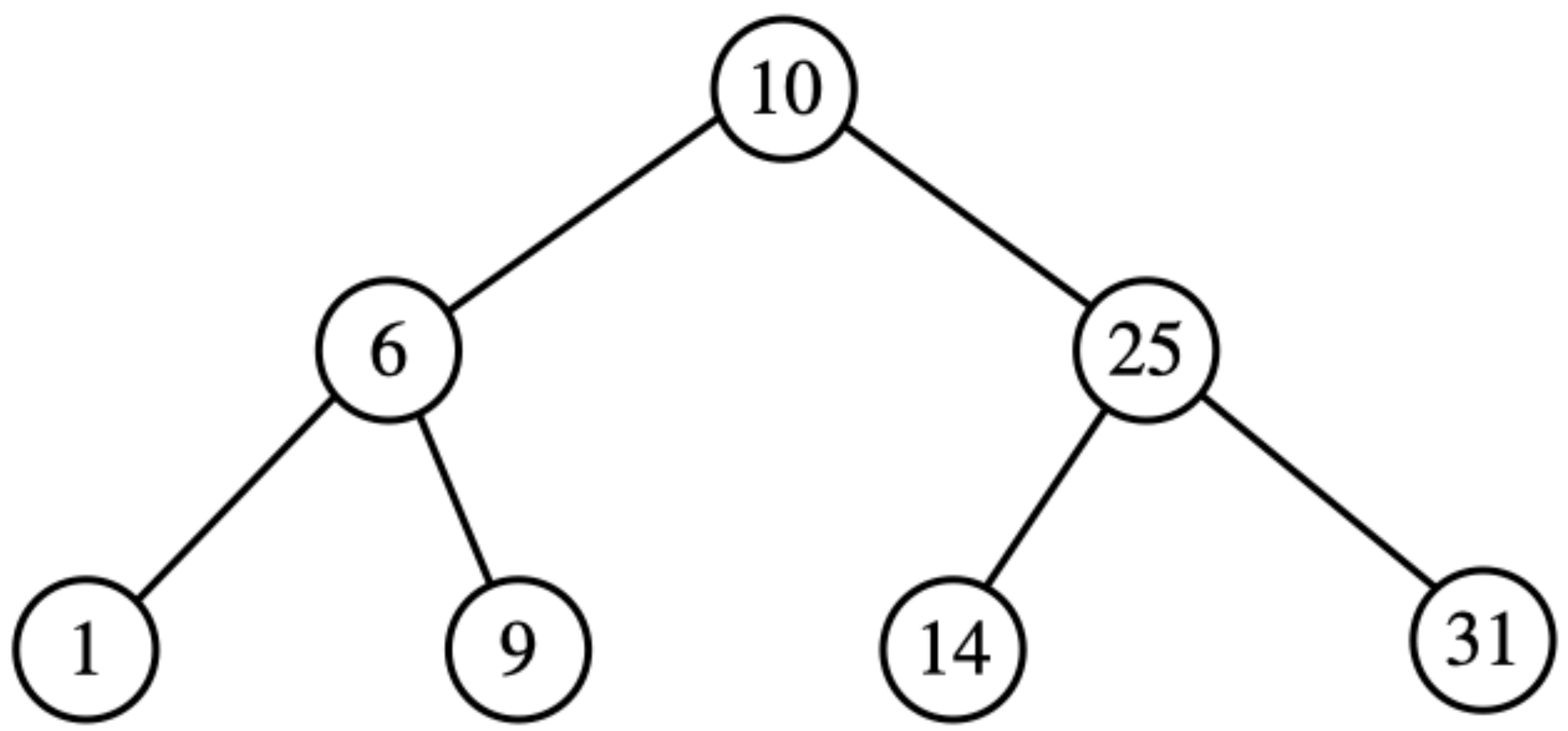

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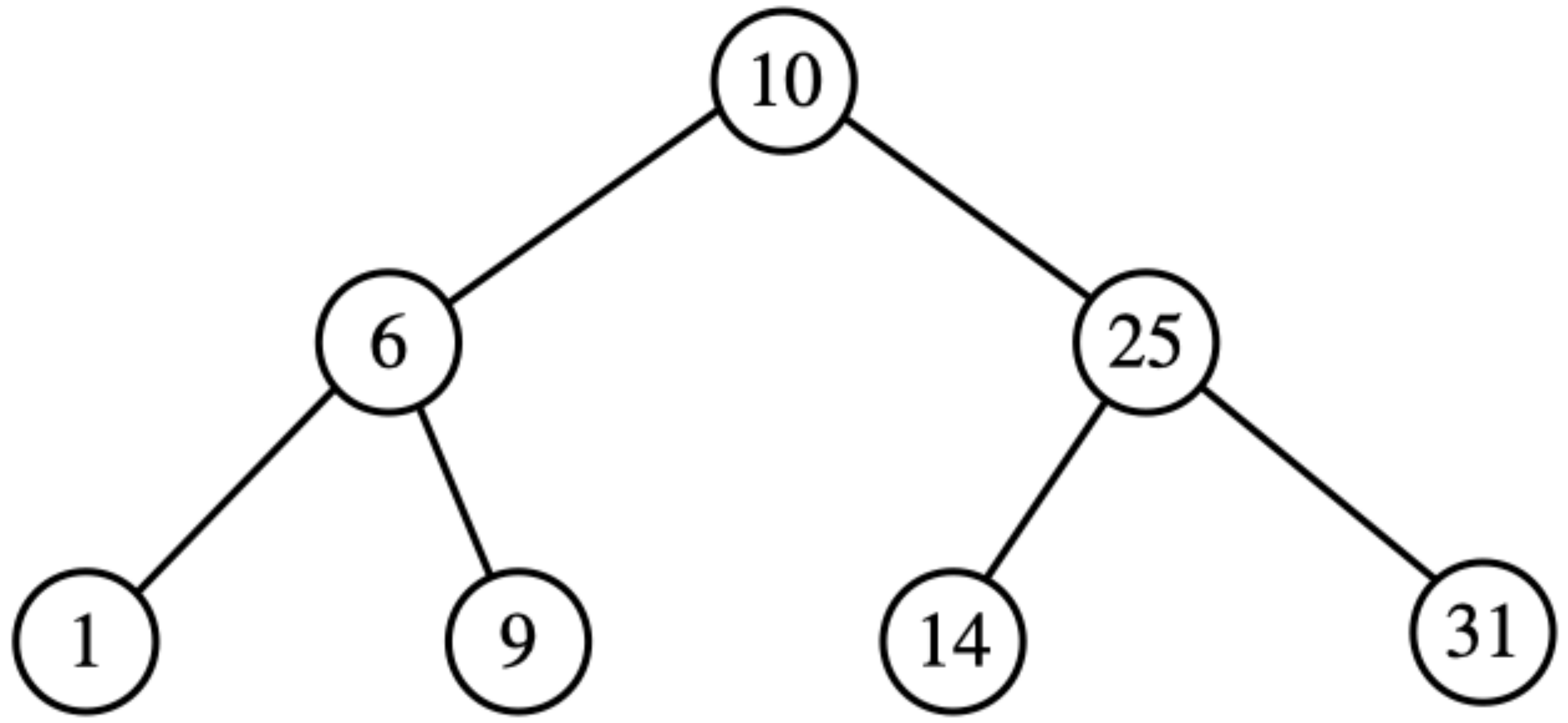

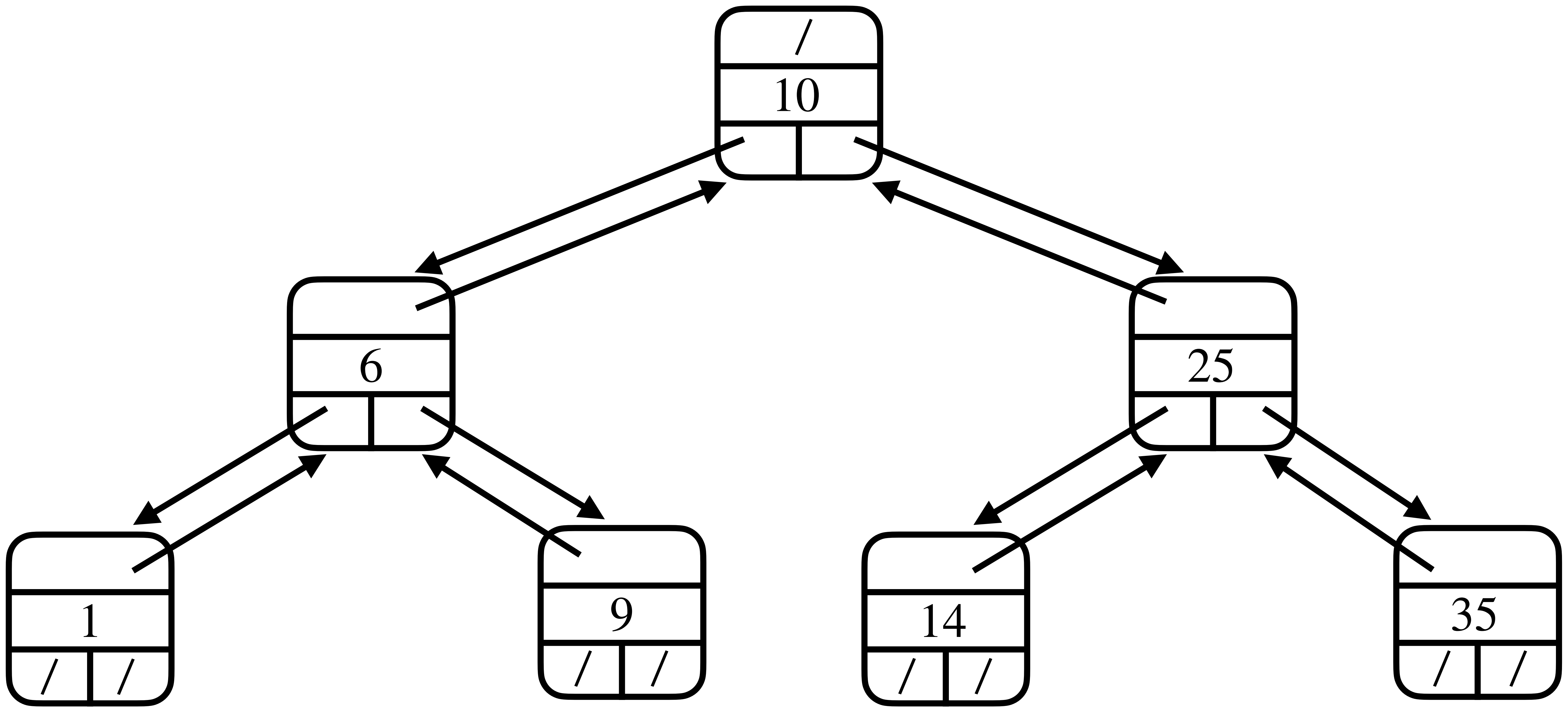

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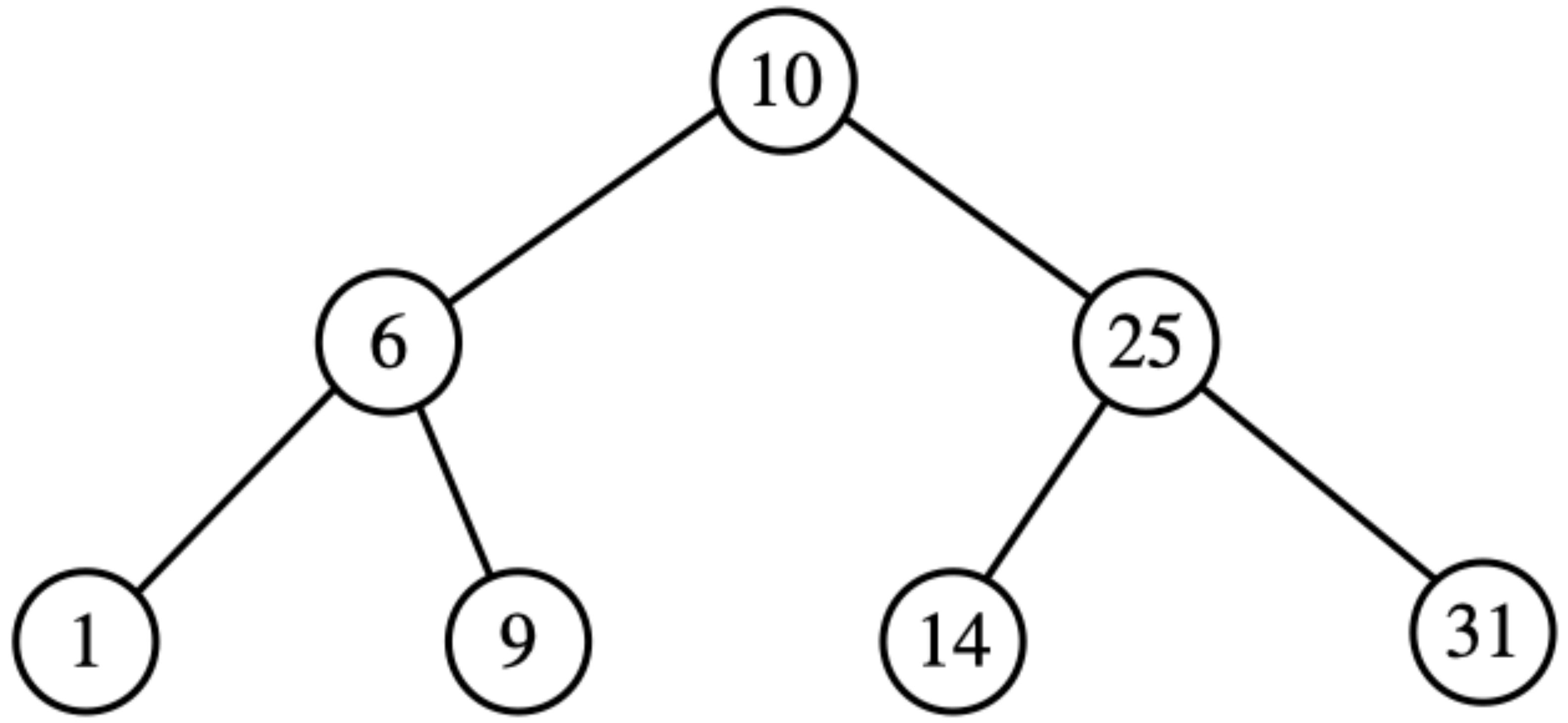

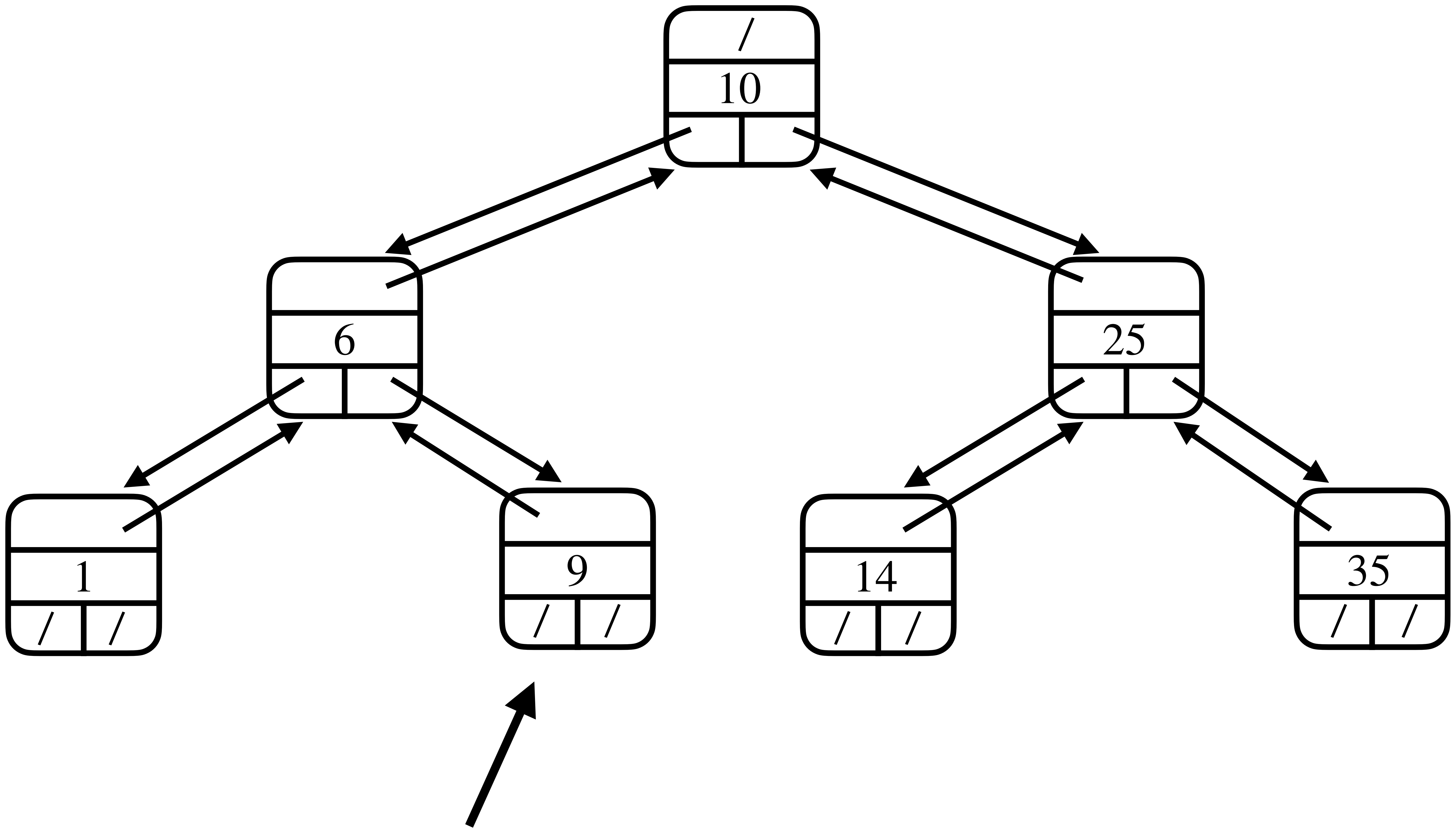

NIL values in the absence of parent, left or right child.

BST: Basic Terminology

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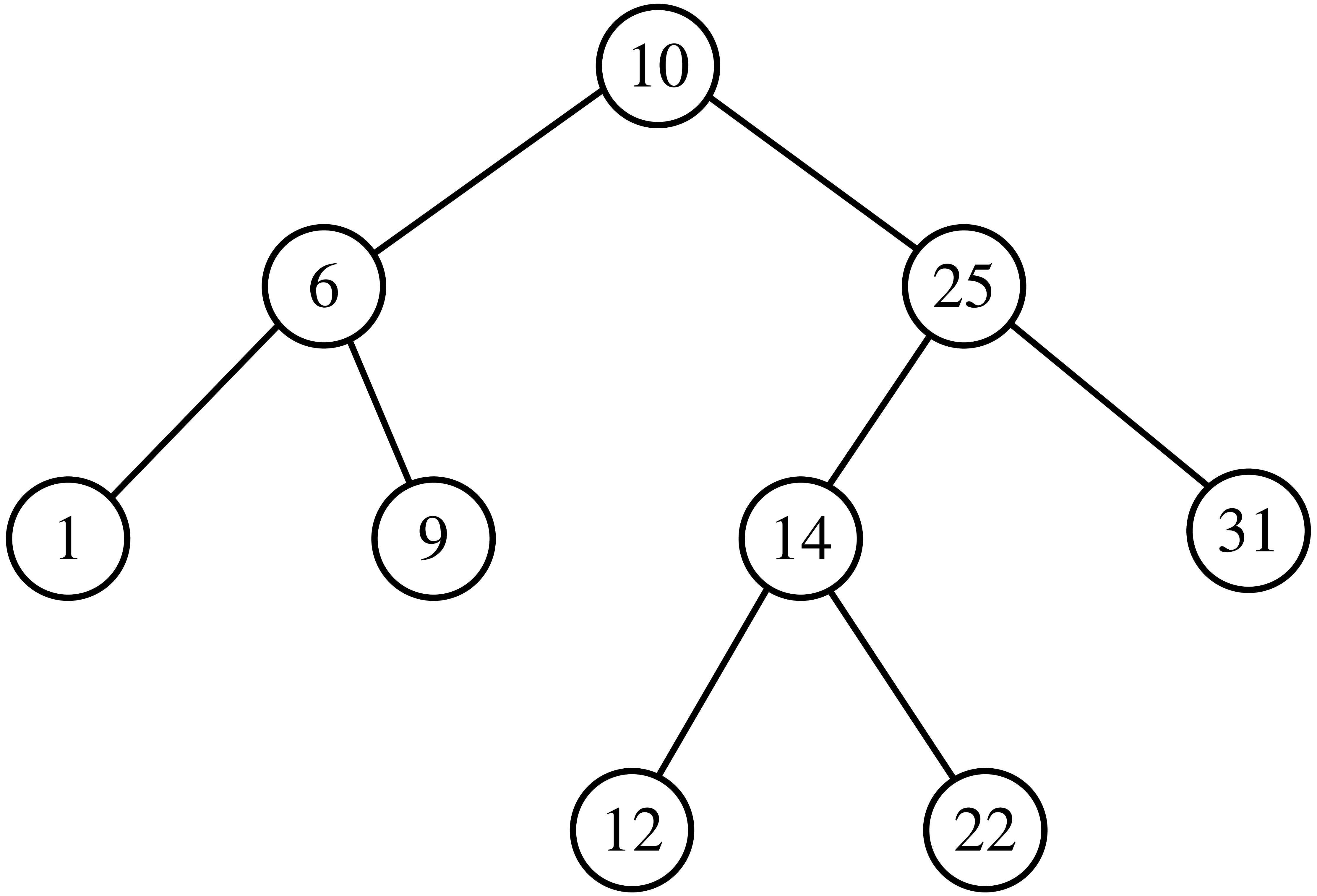

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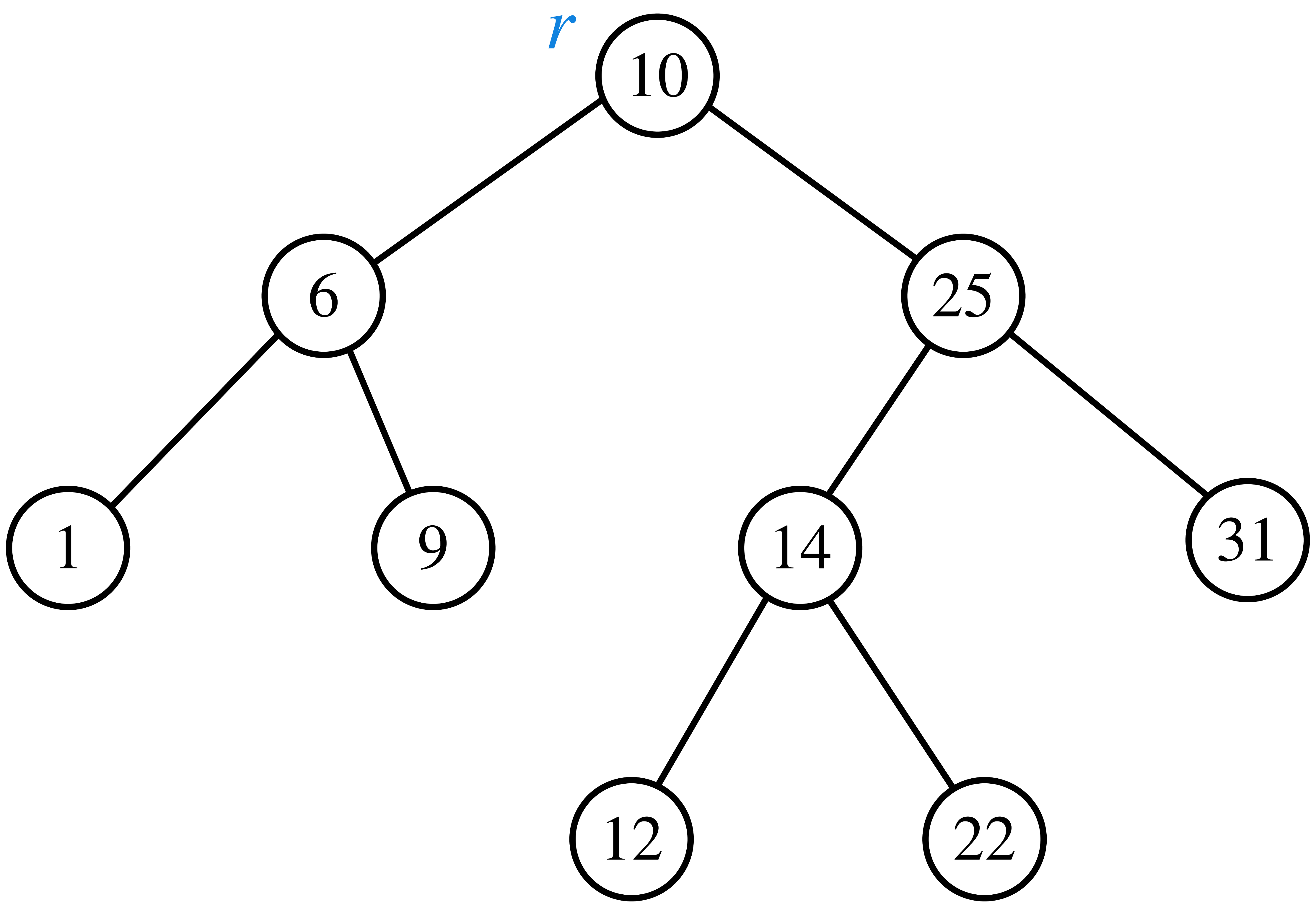

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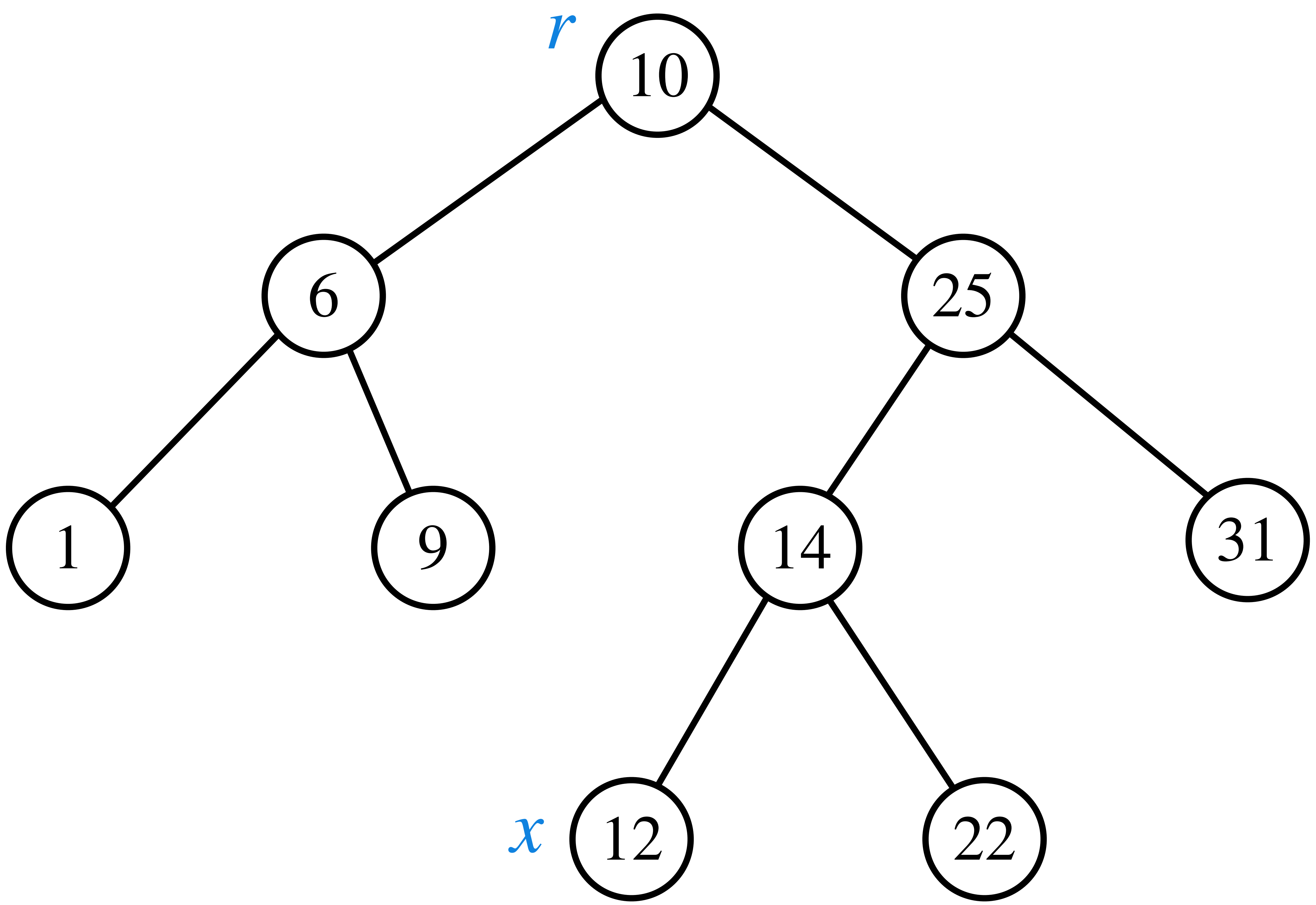

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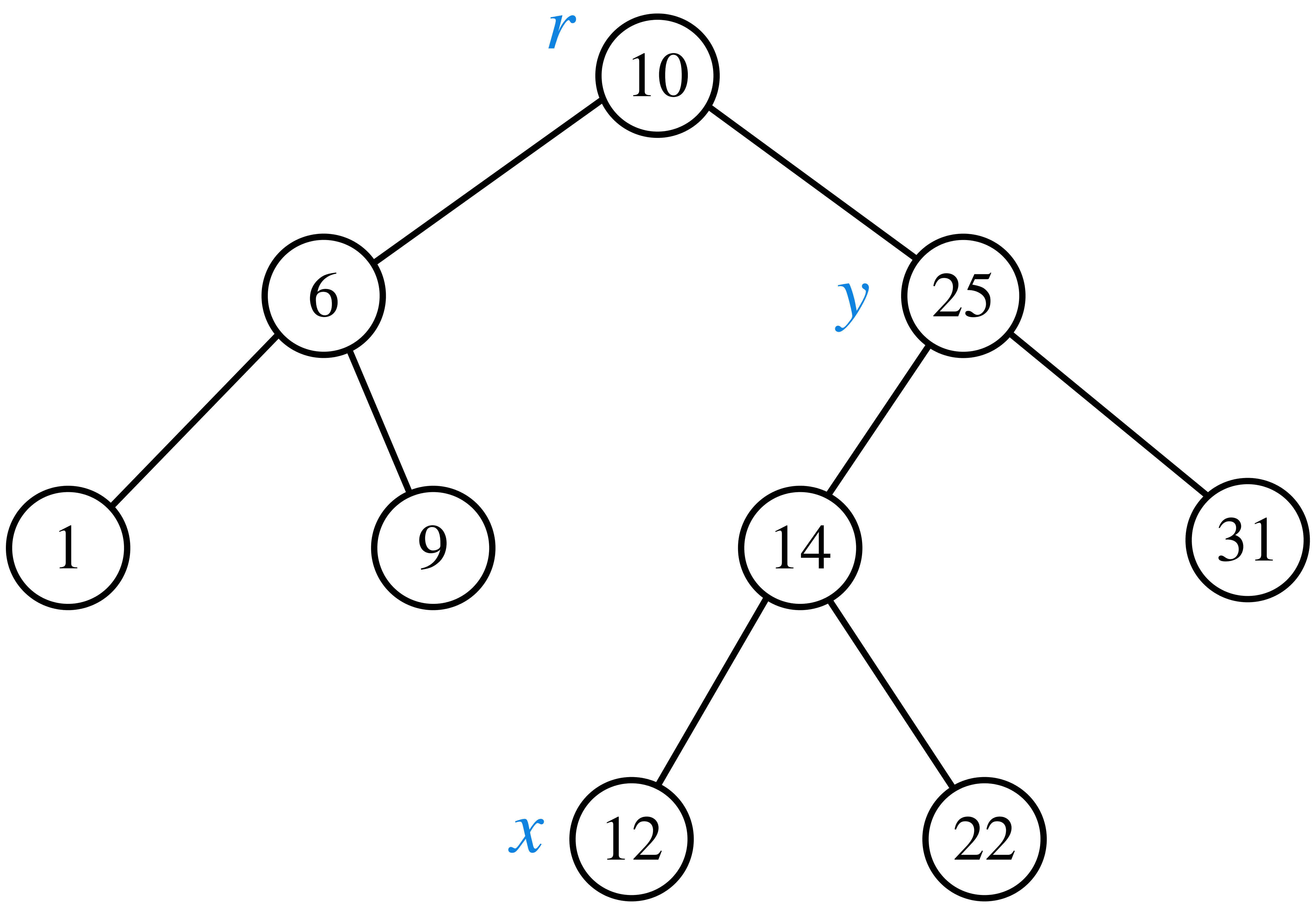

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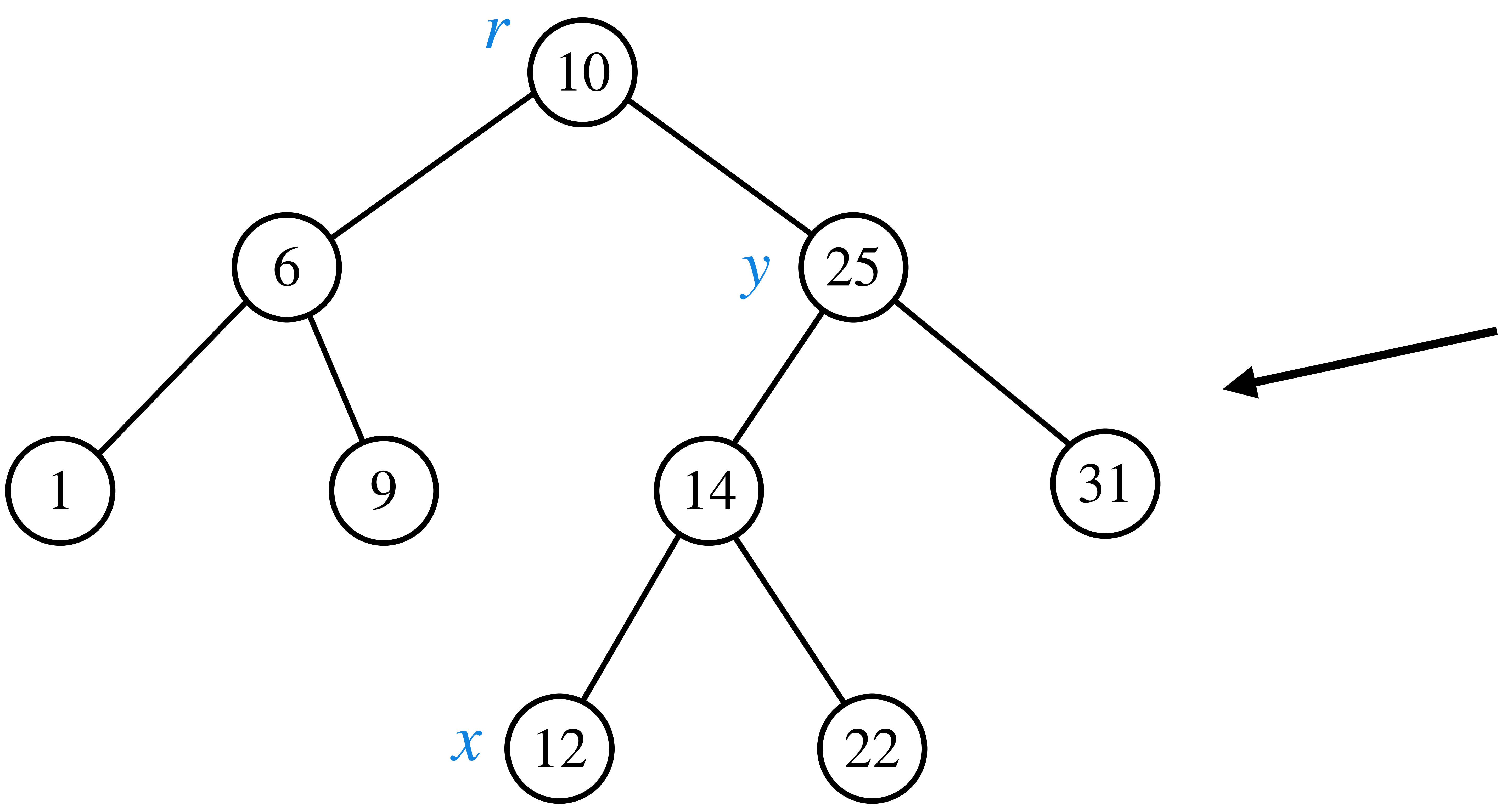

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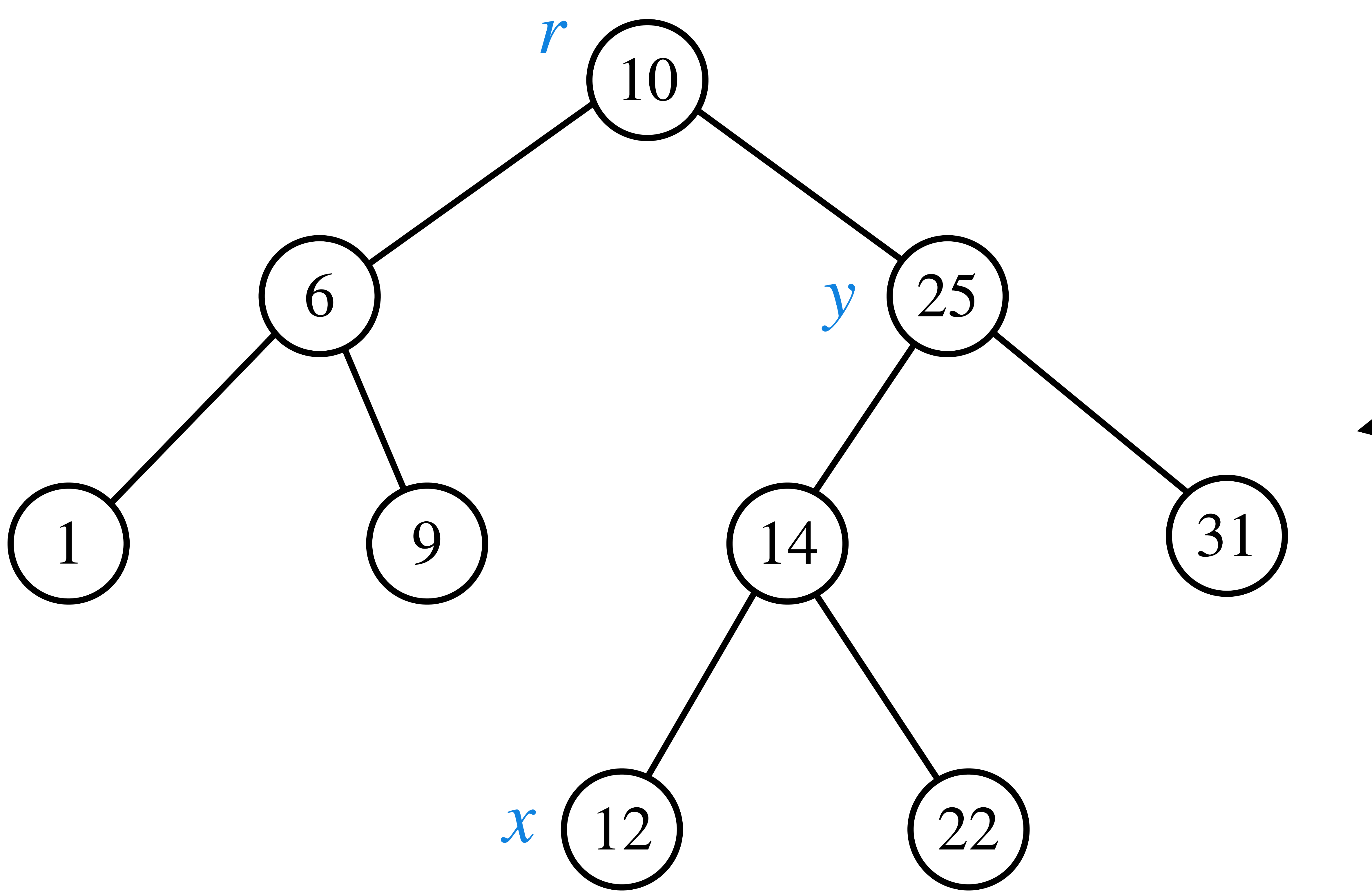

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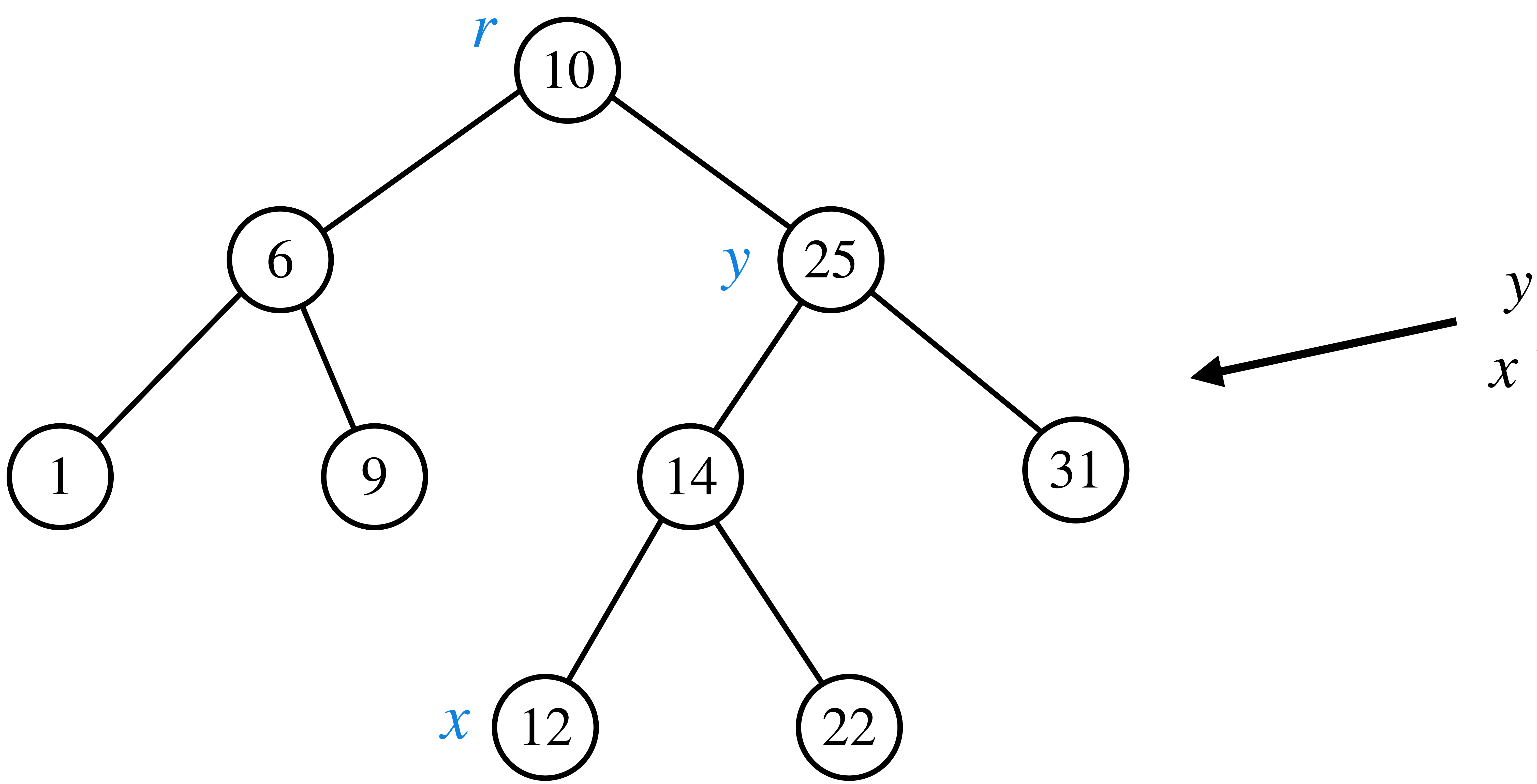

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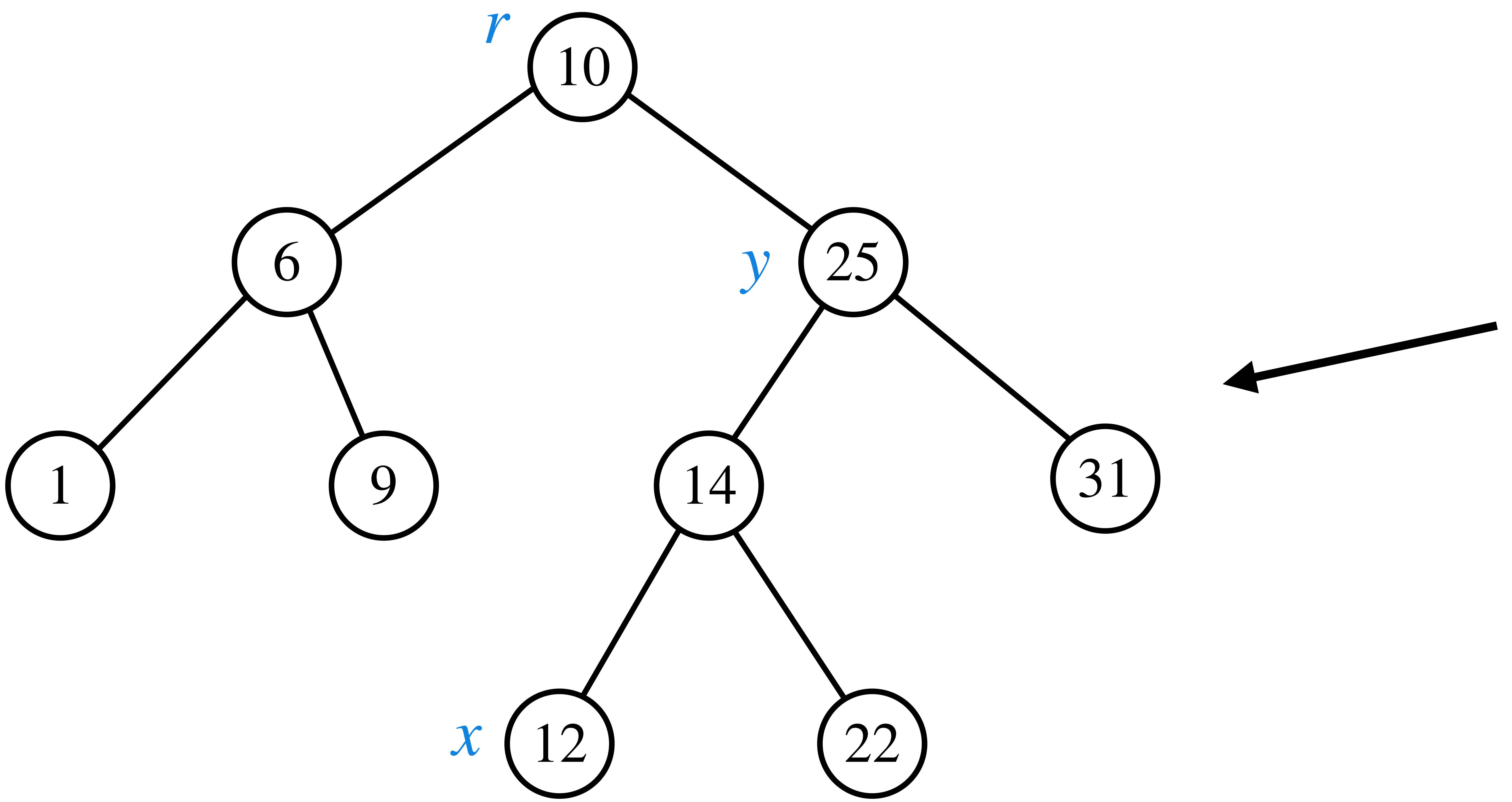

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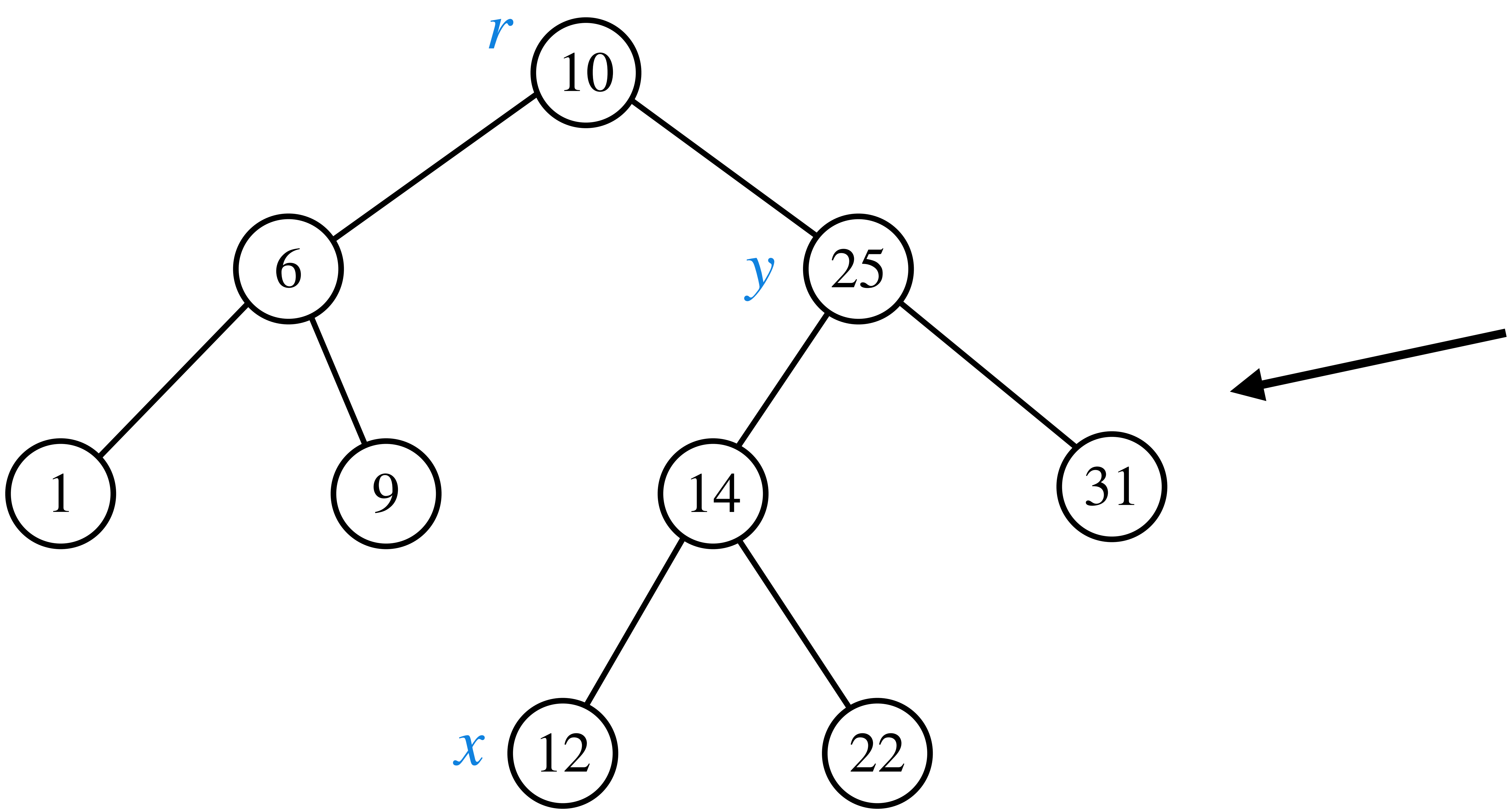

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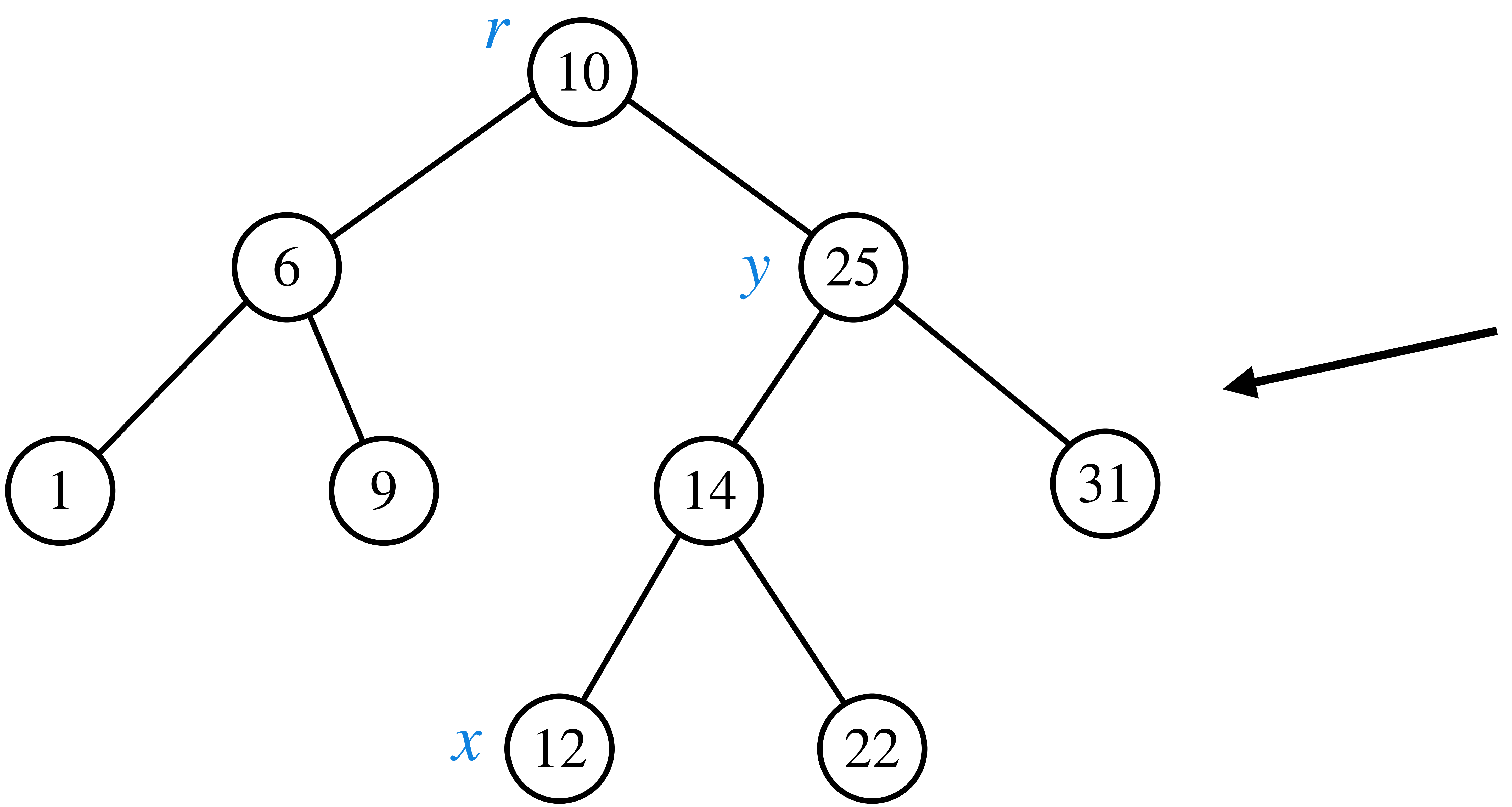

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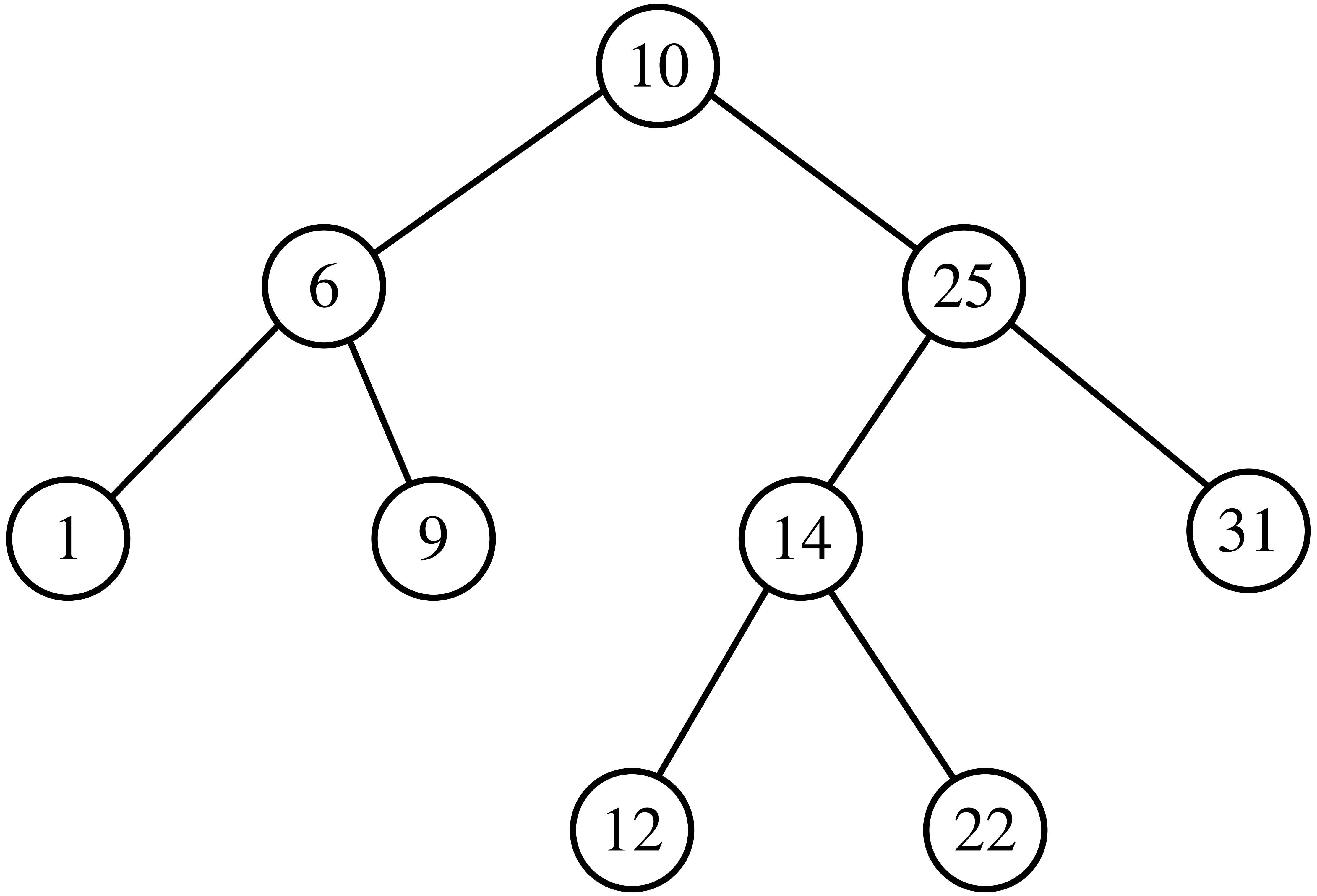

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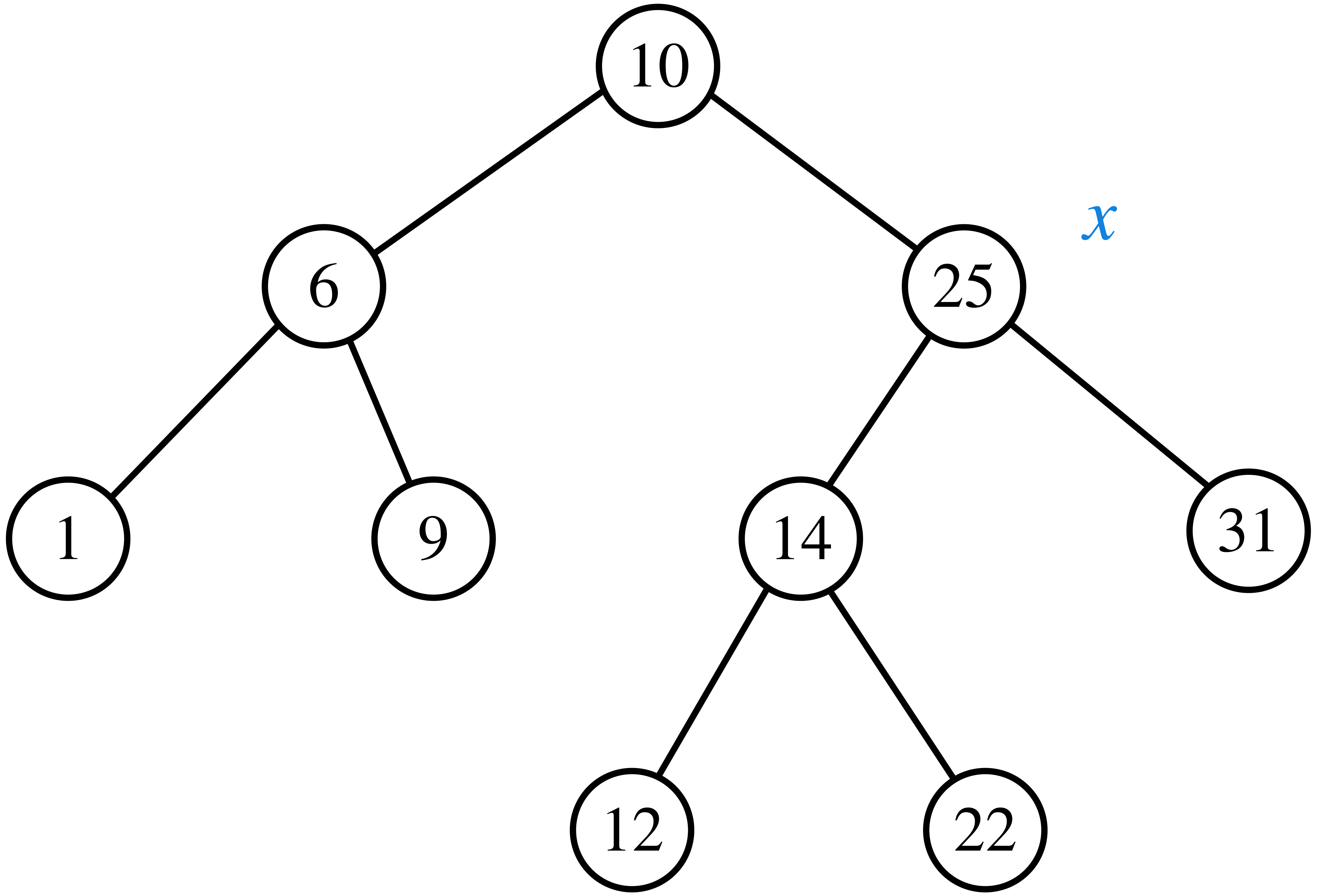

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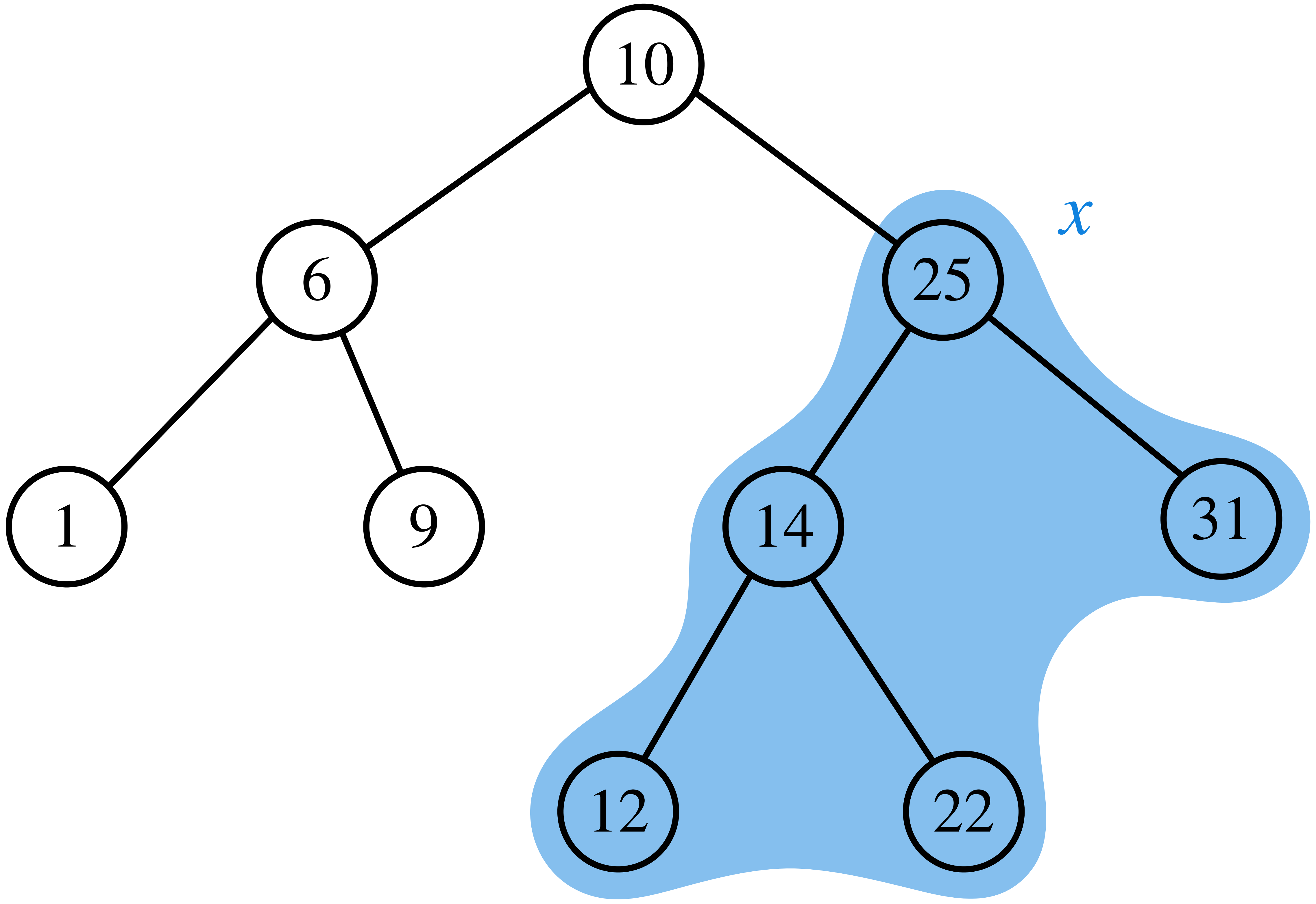

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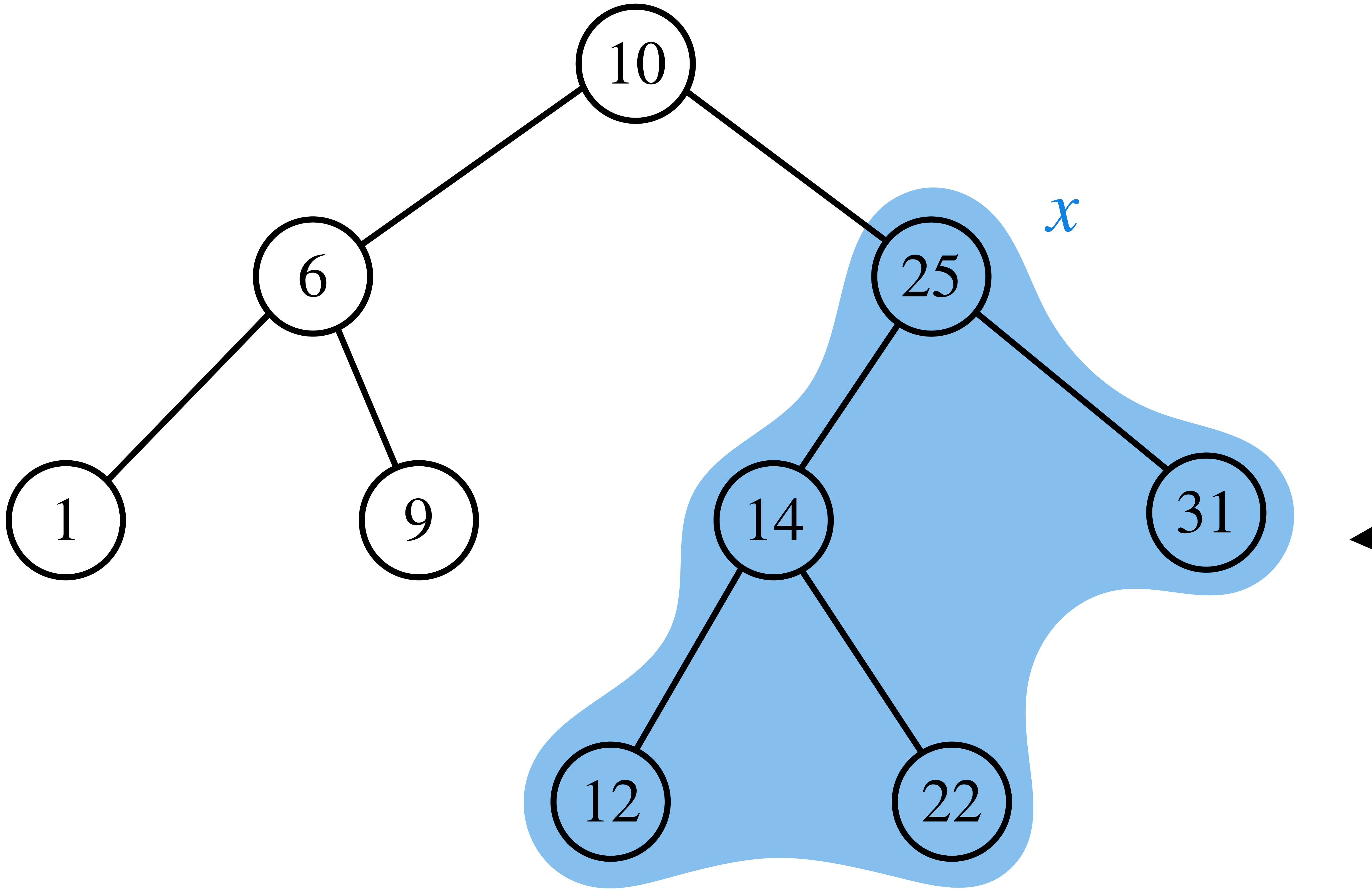
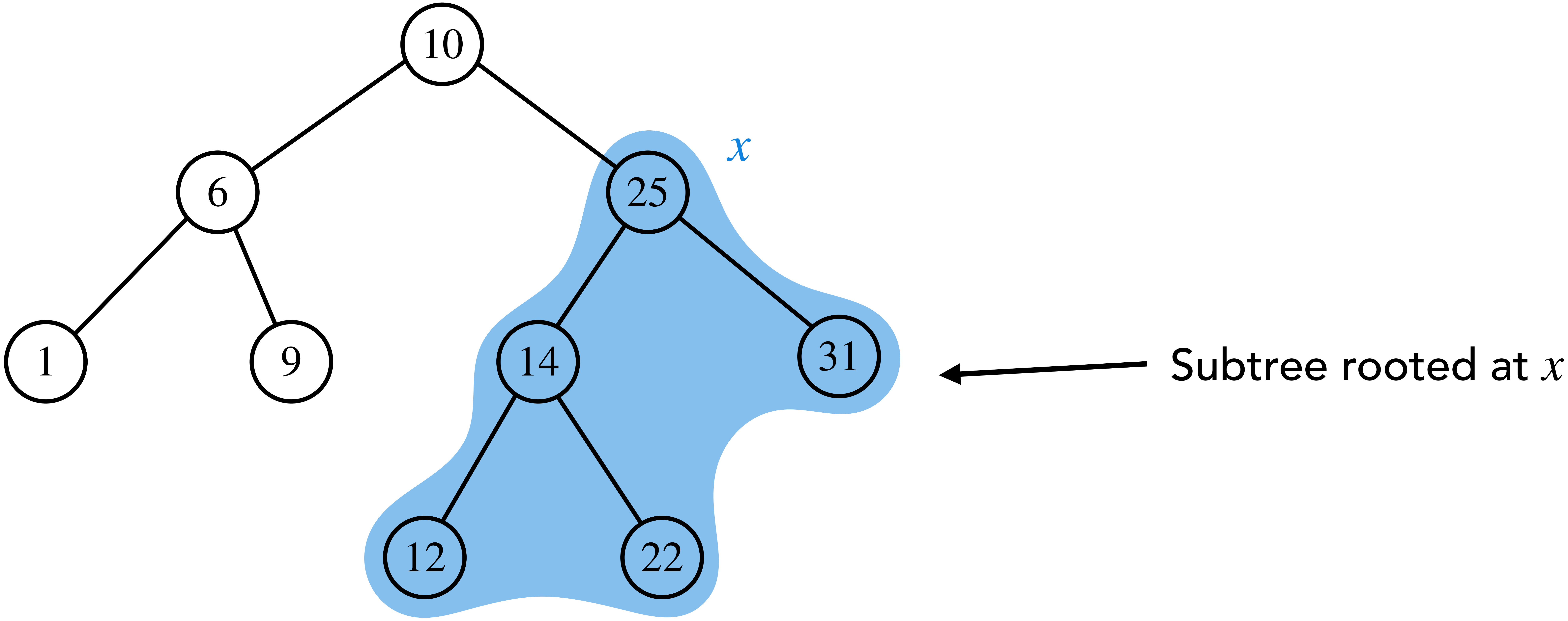


Subtree rooted at x

BST: Basic Terminology

Defn: Subtree rooted at x is the tree containing only descendants of x.



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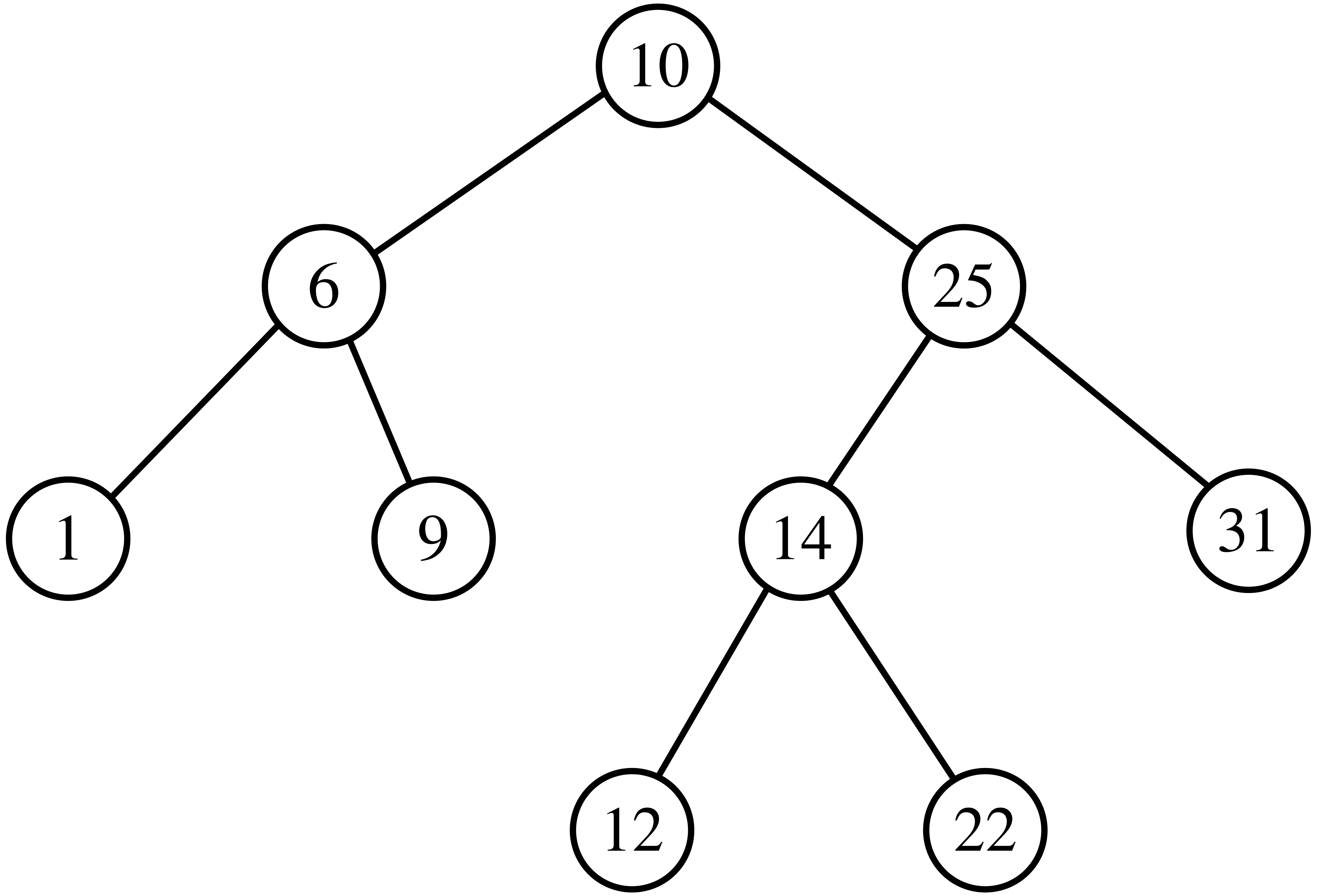

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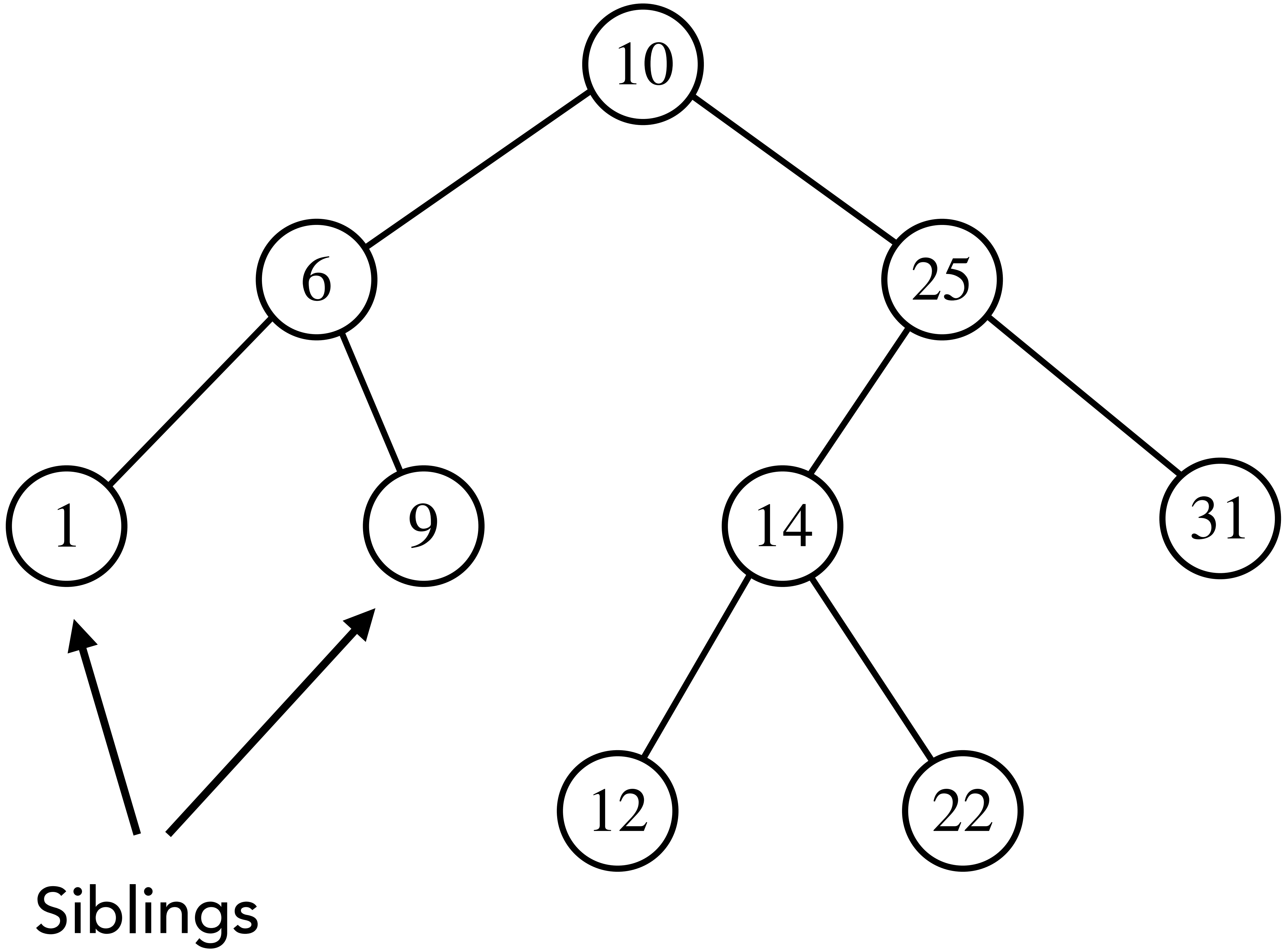

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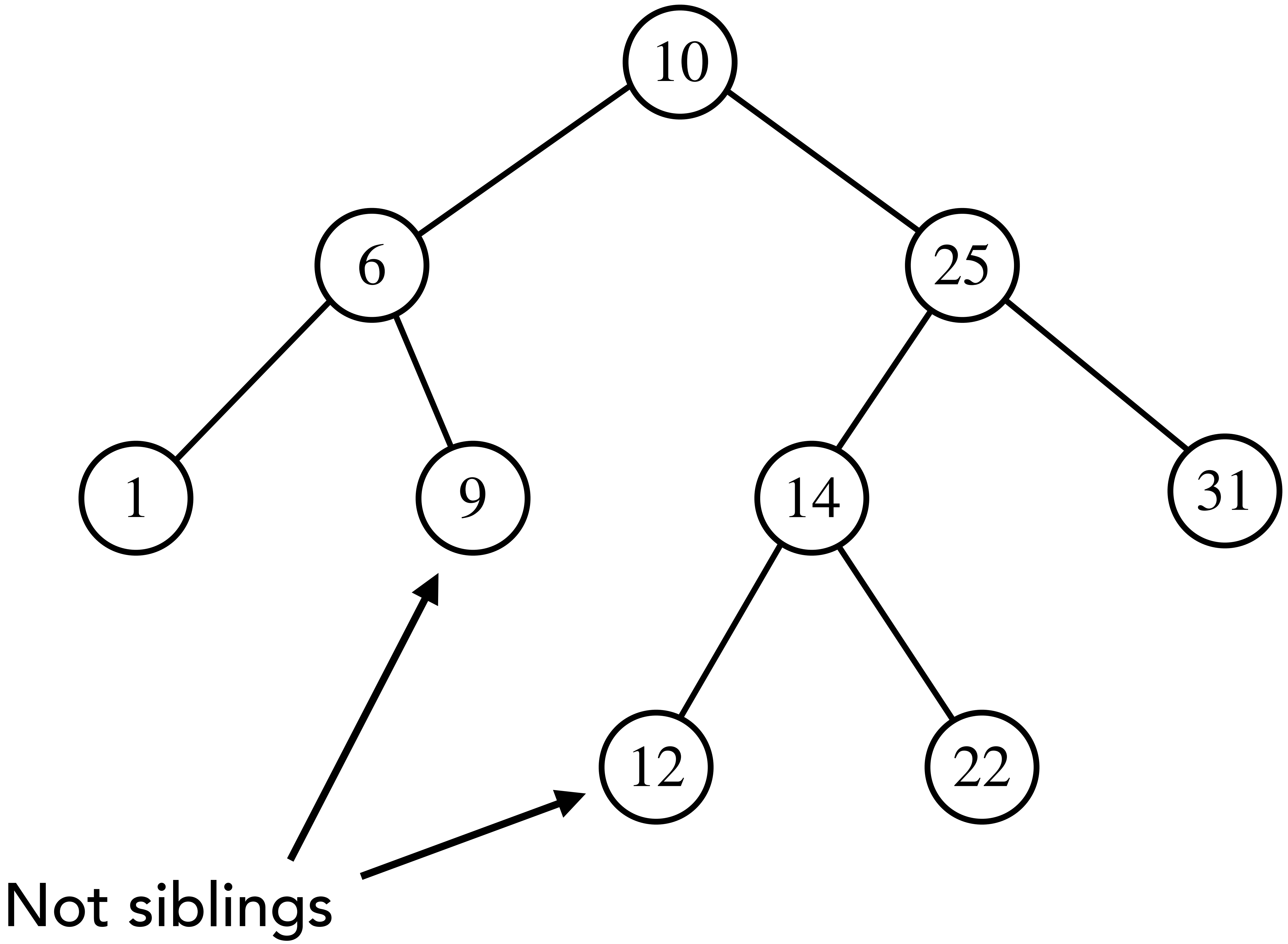

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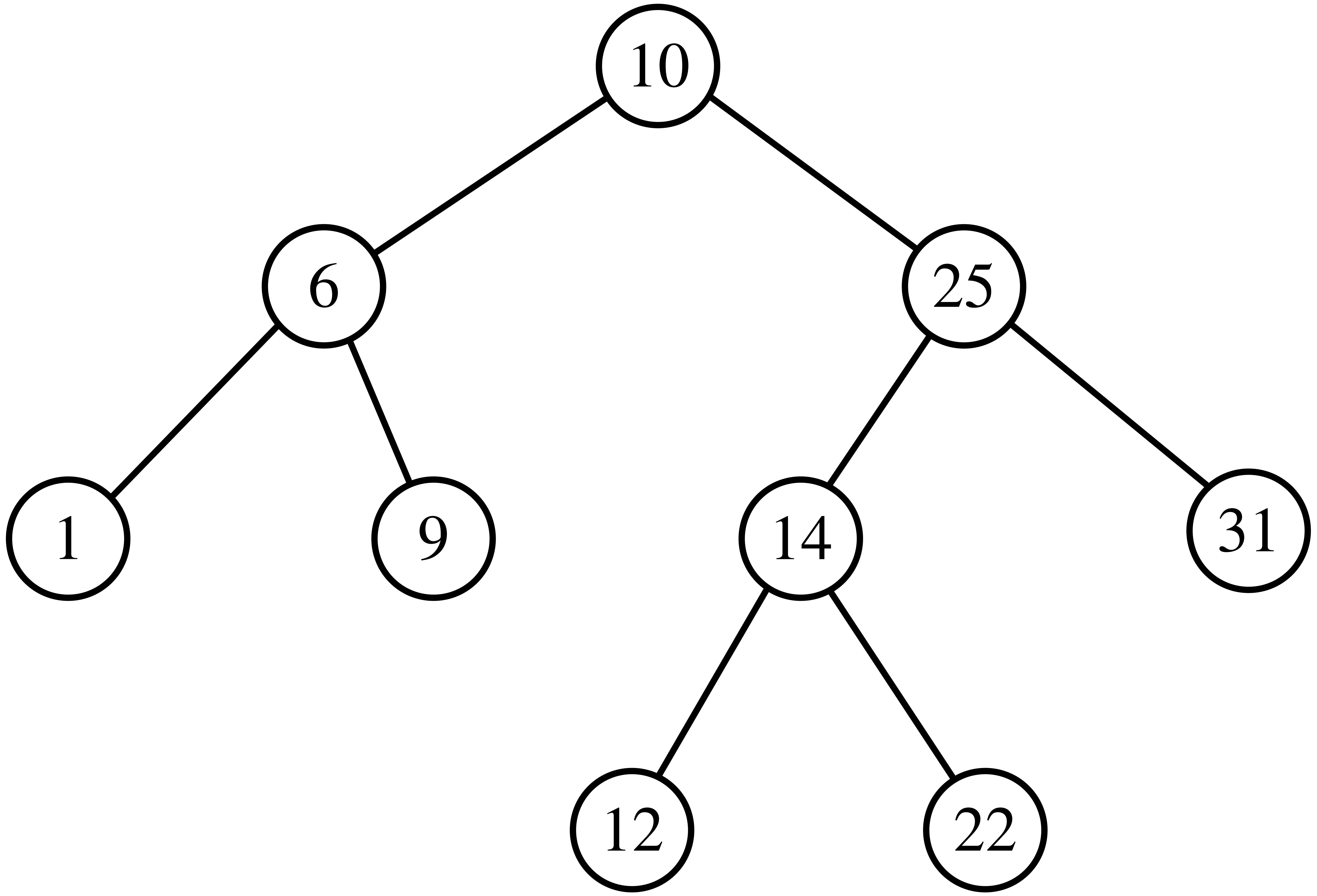

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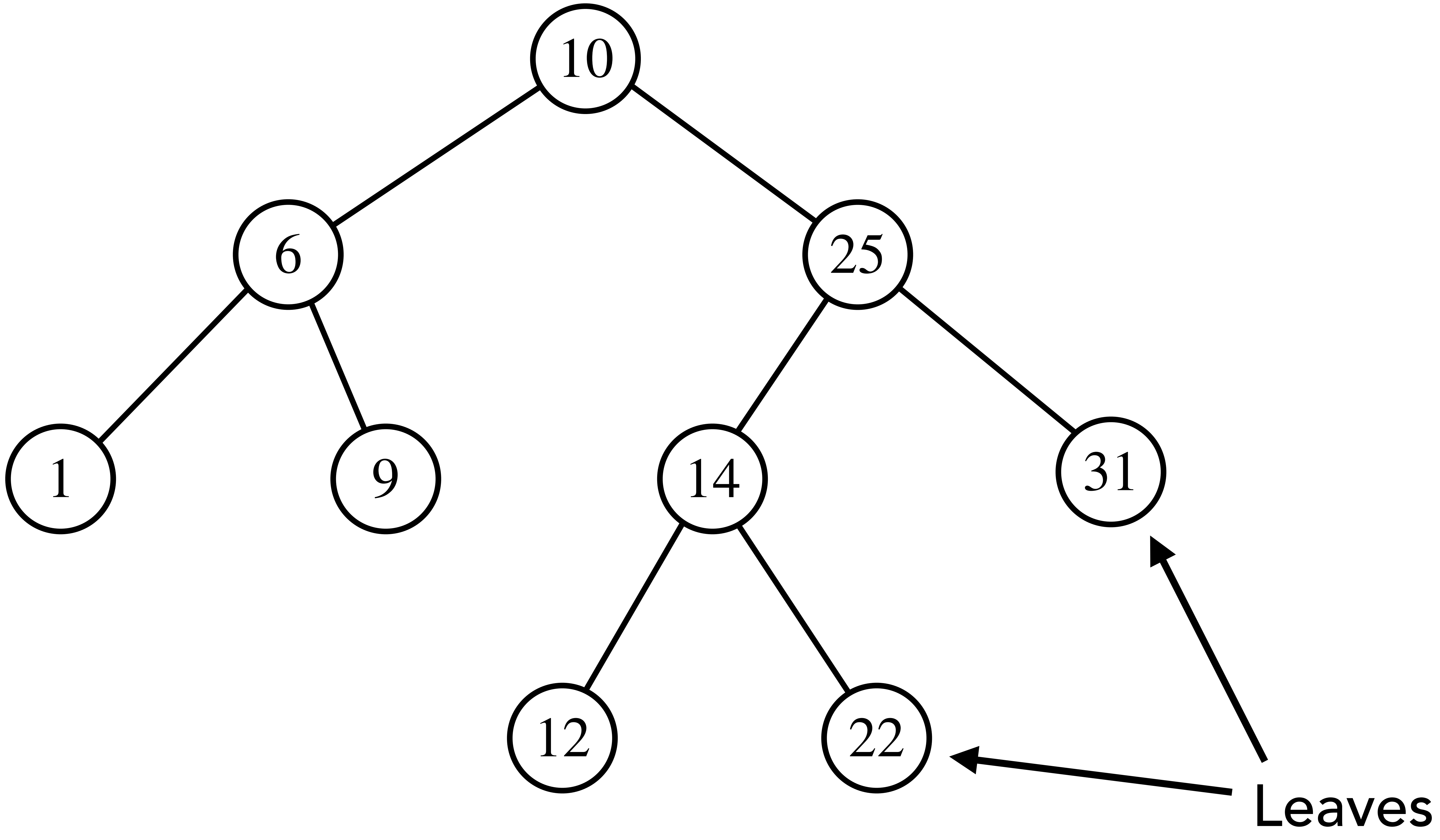

BST: Basic Terminology

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Defn: The height of a node in a tree

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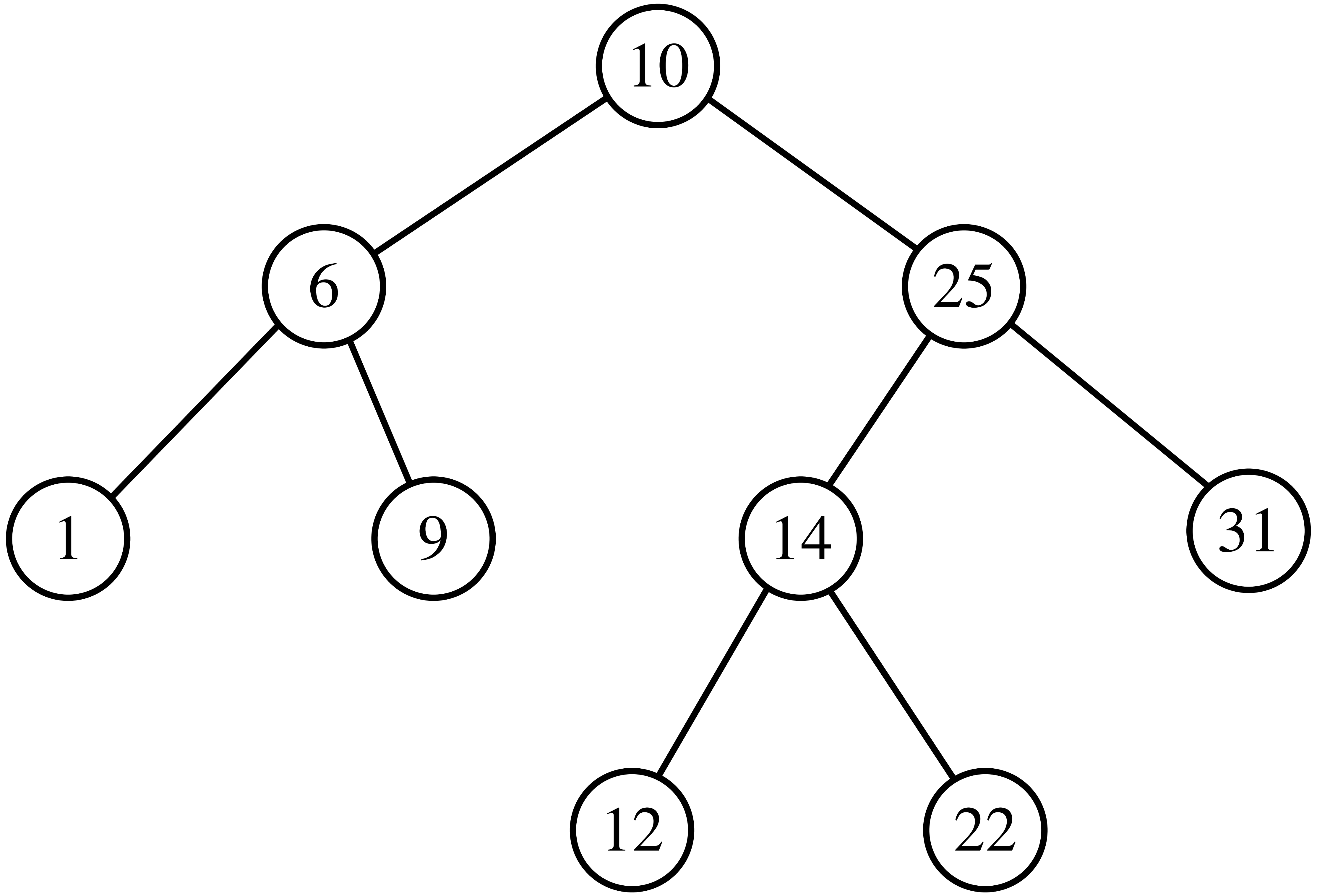

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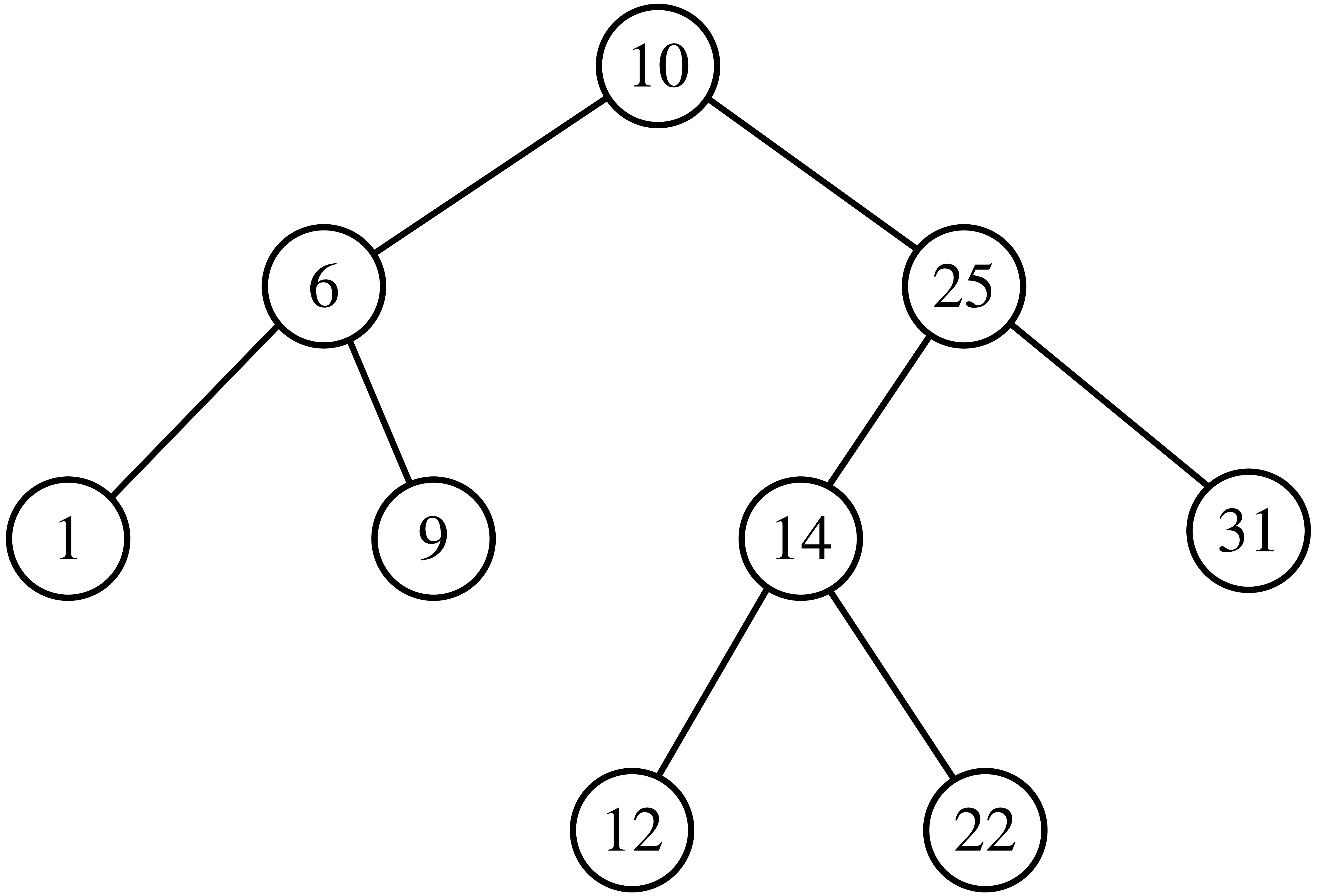

Height of all the nodes

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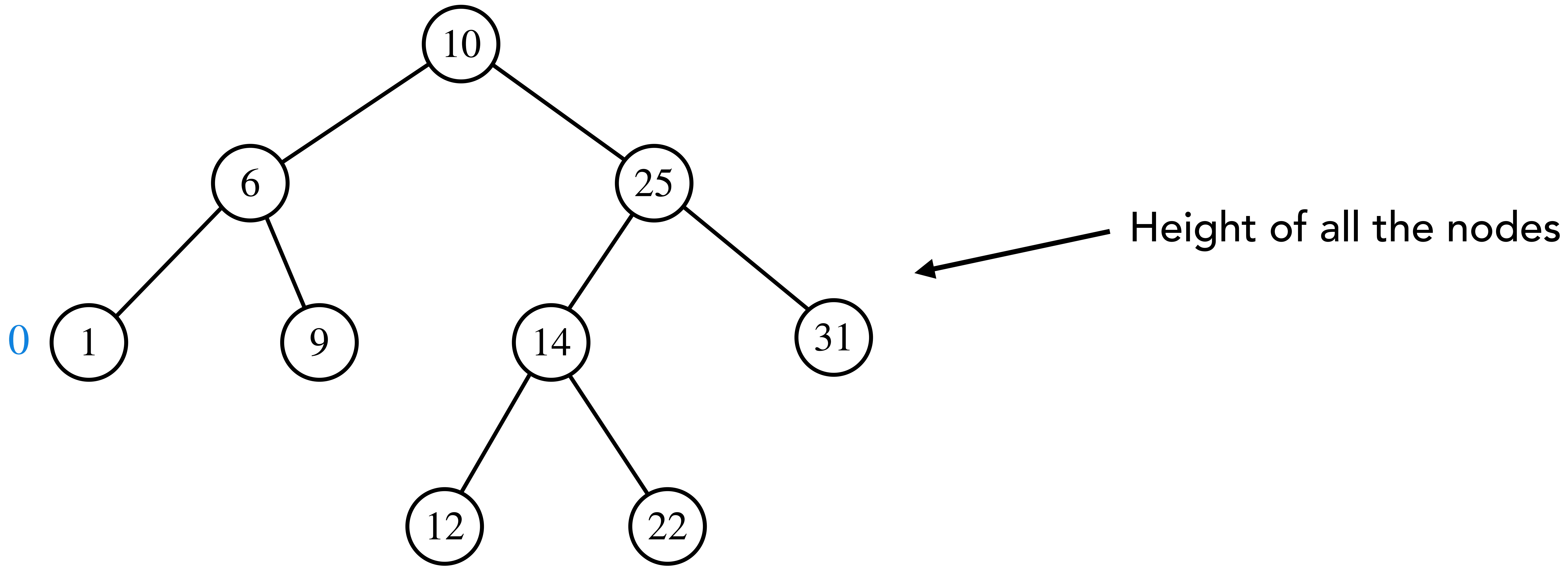

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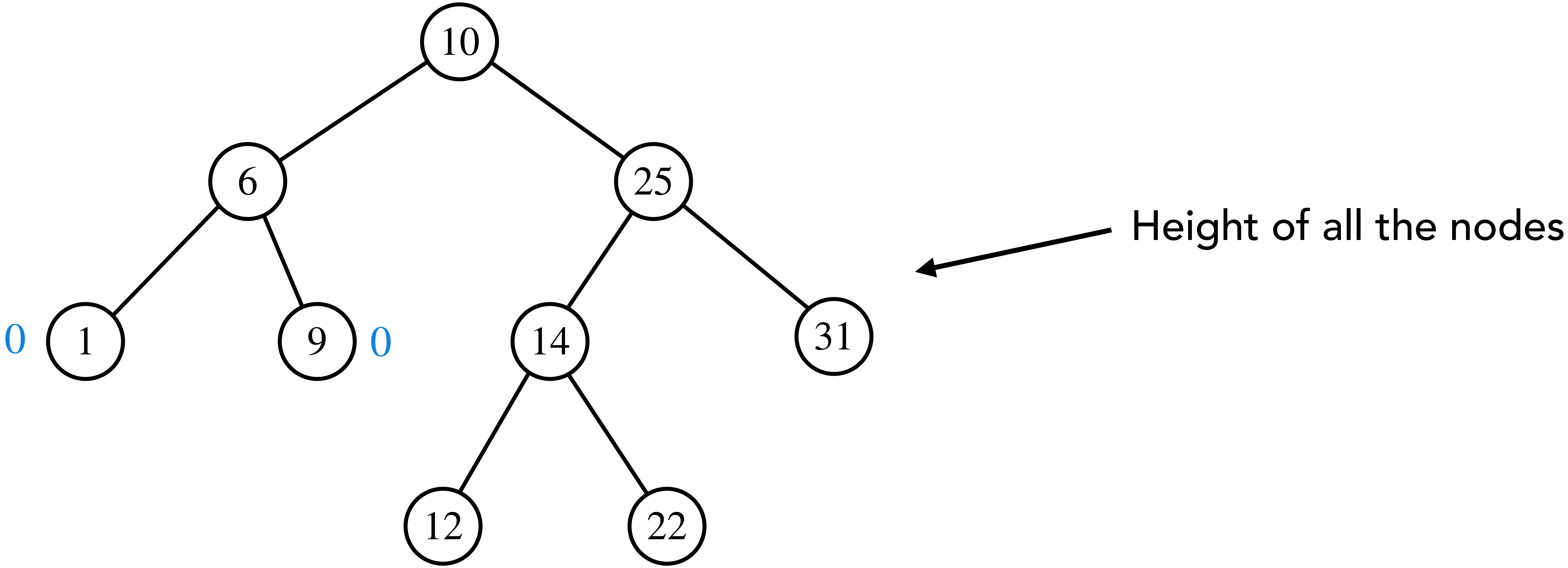

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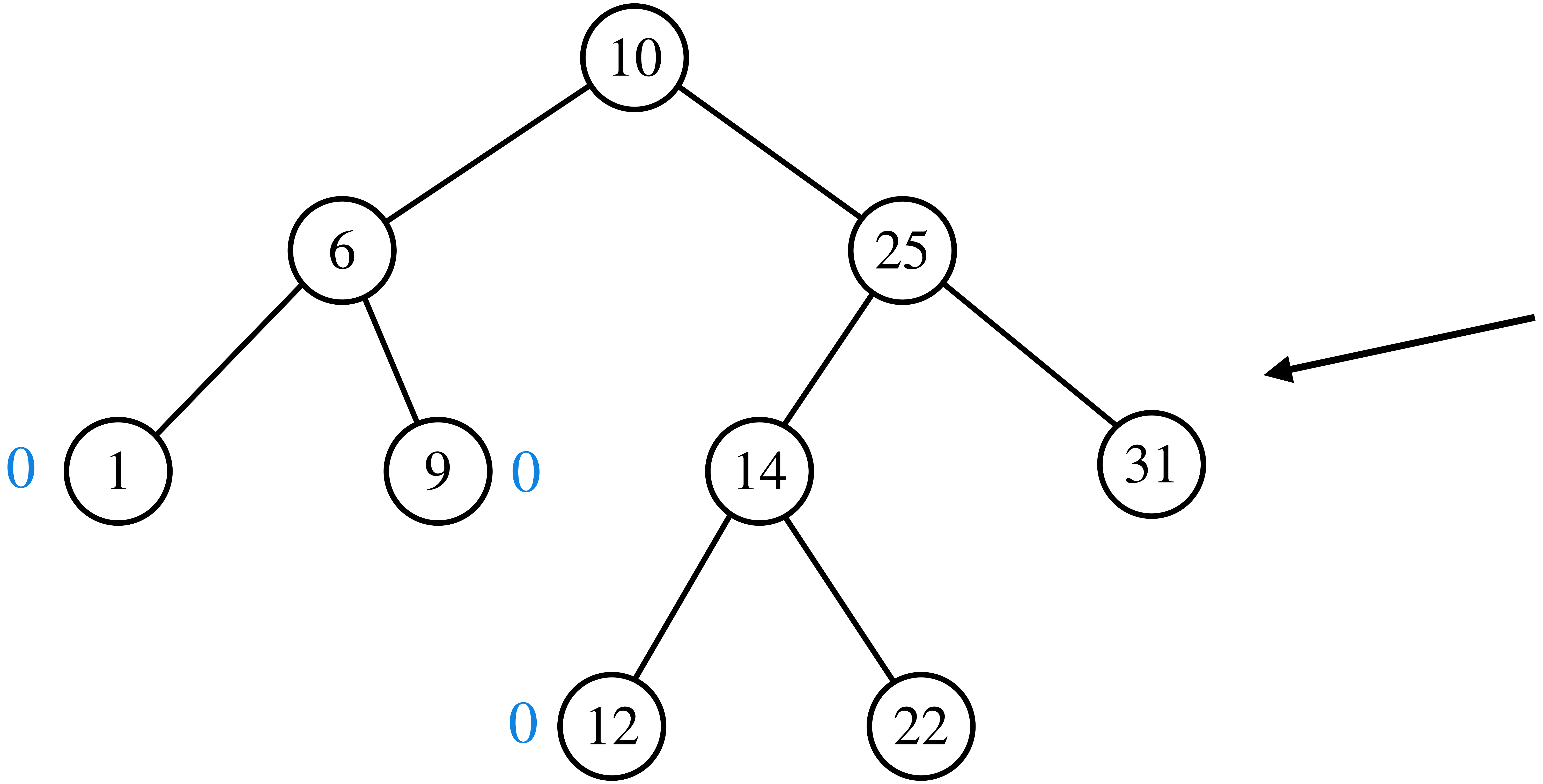

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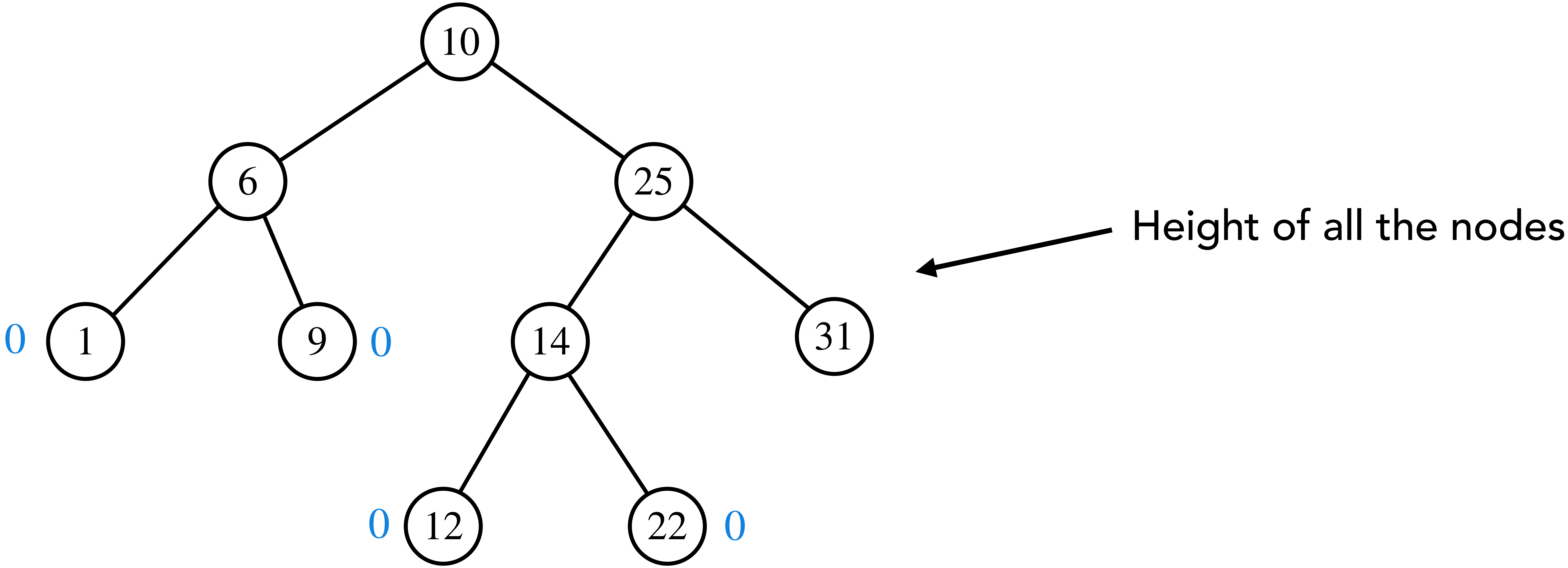

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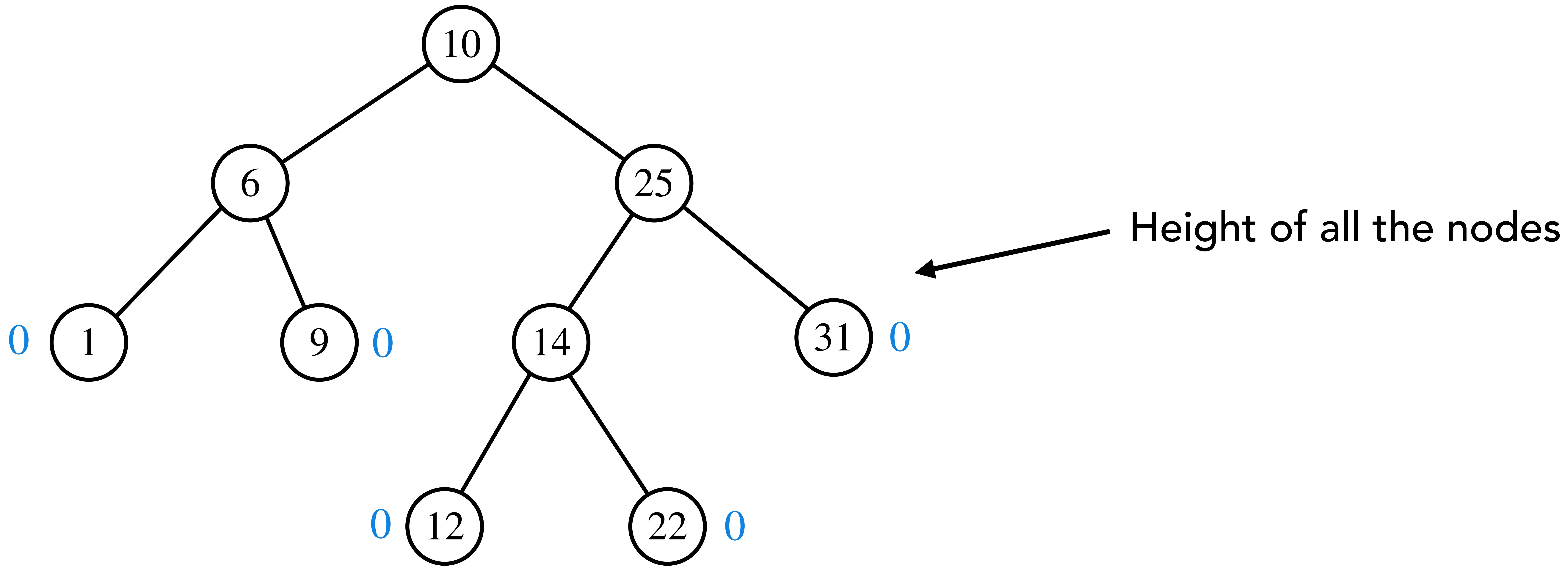

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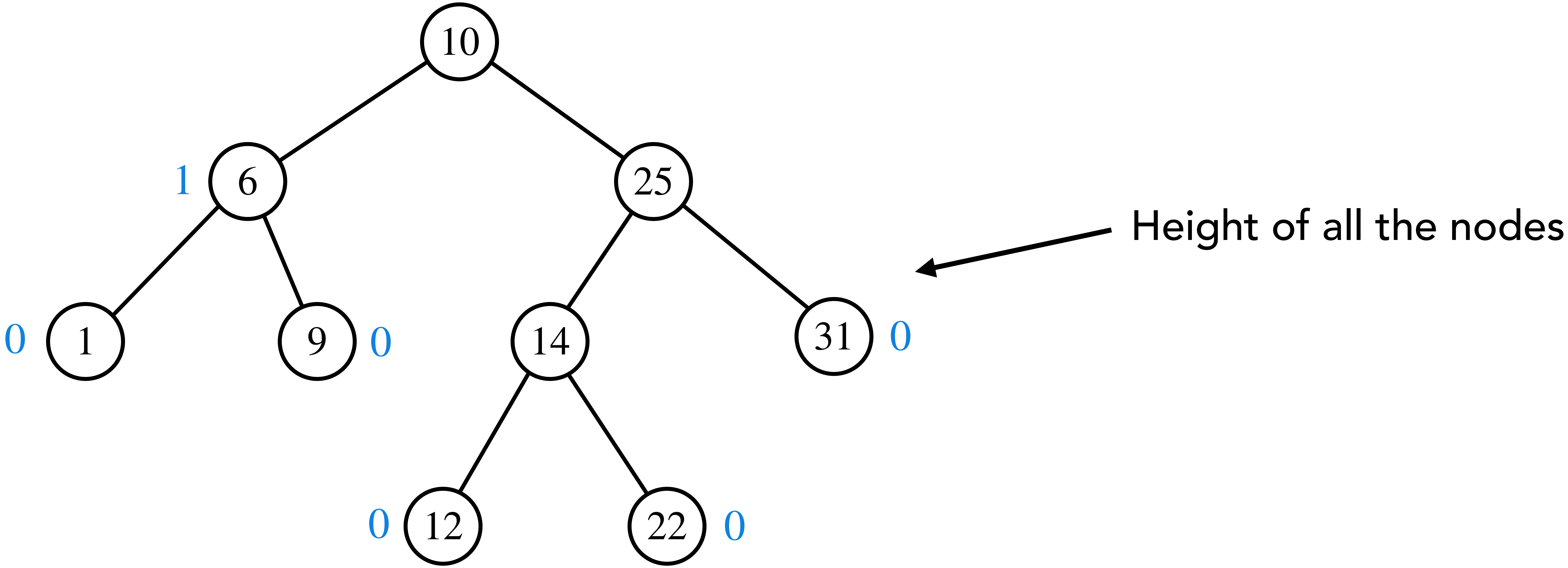

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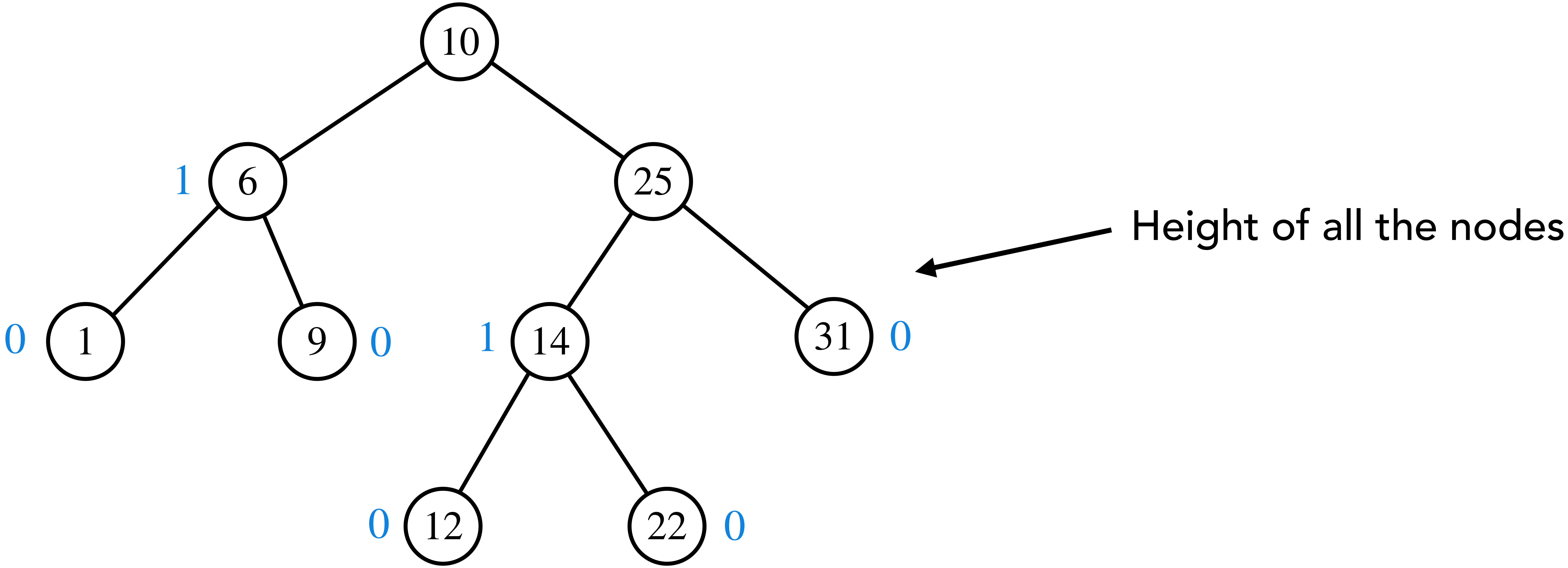

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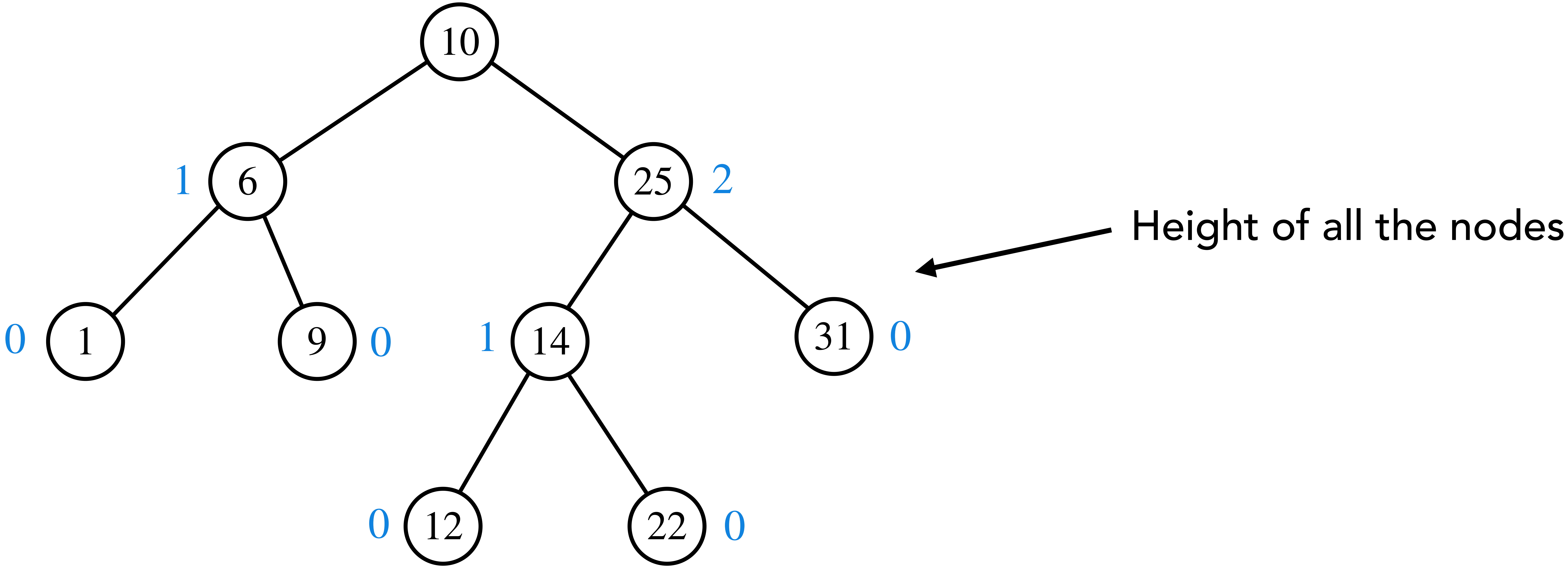

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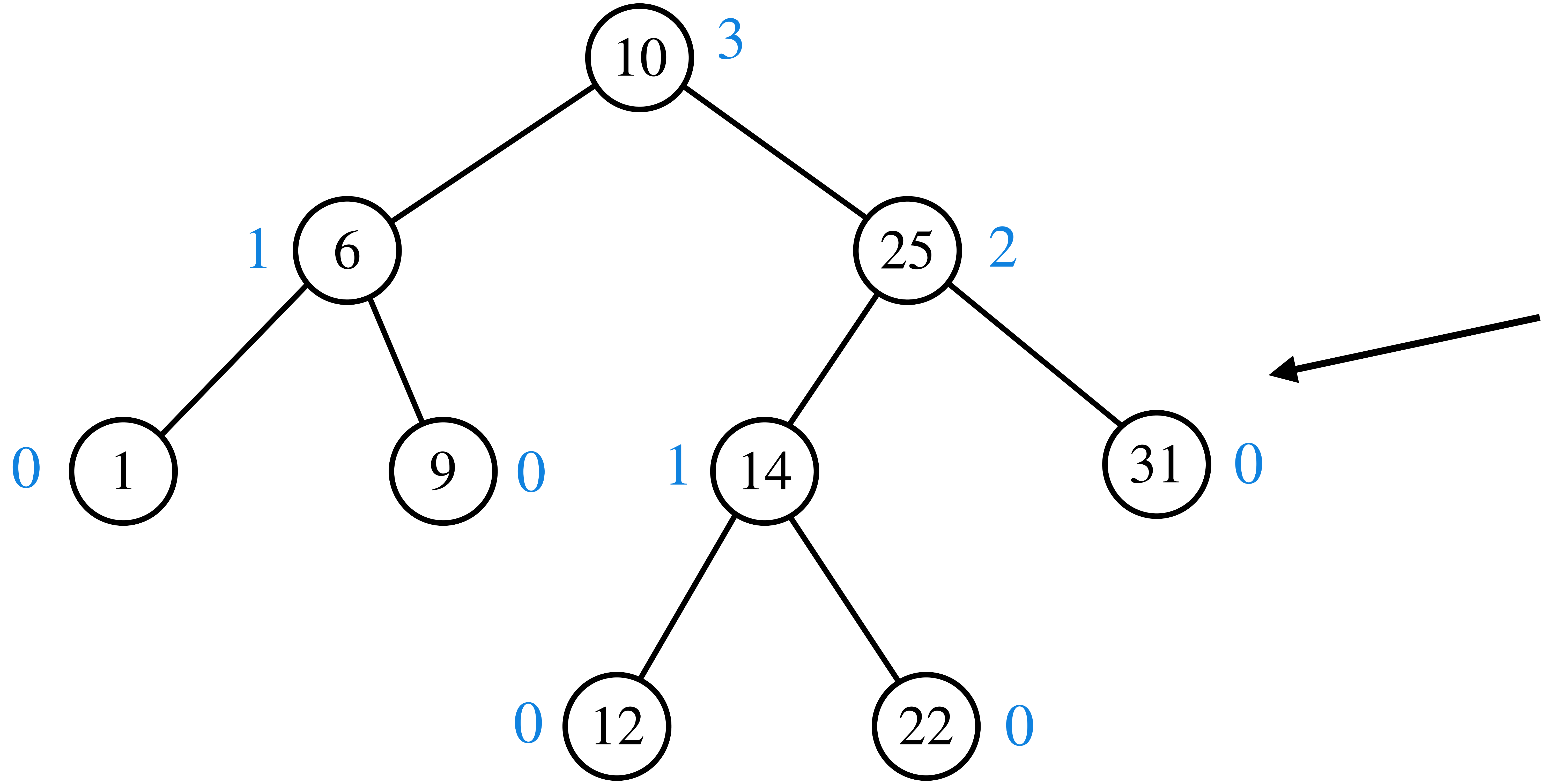

Height of all the nodes

Inorder Tree Walk

Inorder Tree Walk

Inorder-Tree-Walk(x):

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1. **if** $x \neq$ NIL

Inorder Tree Walk

Inorder-Tree-Walk(x)**:**

1. **if** $x \neq$ NIL
2. **Inorder-Tree-Walk**(x . *left*)

Inorder Tree Walk

```
Inorder-Tree-Walk(x):
  if x ≠ NIL
      Inorder-Tree-Walk(x . left)
      print x . key
```

Inorder Tree Walk

Inorder-Tree-Walk(x):

1. **if** $x \neq$ NIL
2. **Inorder-Tree-Walk**($x.left$)
3. **print** $x.key$
4. **Inorder-Tree-Walk**($x.right$)

Inorder Tree Walk

Calling **Inorder-Tree-Walk**($T.root$) will print the keys of the BST T in sorted order.

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Calling **Inorder-Tree-Walk**($T\,.\,root$) will print the keys of the BST T in sorted order.

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if $x \neq$ NIL
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    Inorder-Tree-Walk($x\,.\,right$)

Proof of Correctness: We will prove it using induction on the number of nodes in the tree.